Motion Encoding and Decoding in the Retina

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Retina: *Gateway to the Visual System*

- The sensor which collects the visual inputs in animals
- Efficiently collects and encodes visual inputs
- Pre-processing visual information prior to more deeper parts of, such as the visual cortex, of the brain
- Object of study from the earliest neuroscientists, such as Ramón y Cajal (1852 – 1934) - father of modern neuroscience
- A popular area of study in modern neuroscience including computational neuroscience
- Studied at different levels and sciences: biology, chemistry, image processing, etc.
- Many applications: cybernetics, prosthetics, etc.

Main Problems

- **Modeling:**

  Develop a model of retinal cells – Bio-realistic & robust

- **Encoding:**

  Characterizing motion encoding properties of retinal cells

- **Decoding:**

  Given retinal responses due to a known class of motion, how can the motion parameters can be recovered?

Challenges

- **Modeling:**

  Many properties reported yet many parameters unknown or extremely hard to find from the known observations.

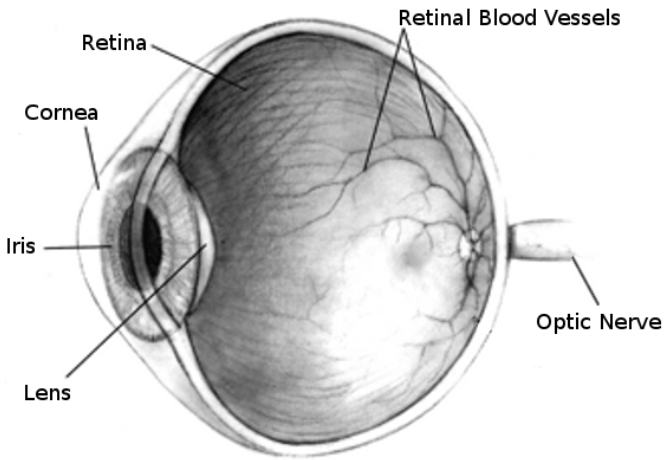
- **Encoding:**

  Large bulk of data generated/recorded. Background noise.

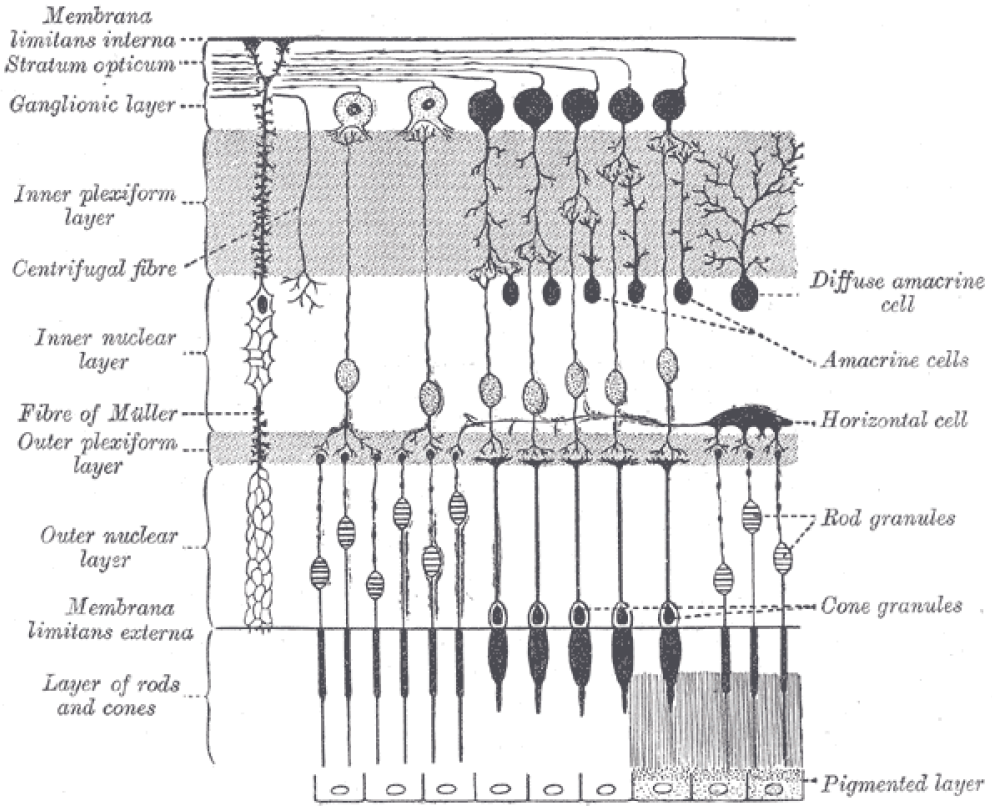
- **Decoding:**

  Large bulk of data generated/recorded. Background noise.

Structure of Retina: *The First "Look"*

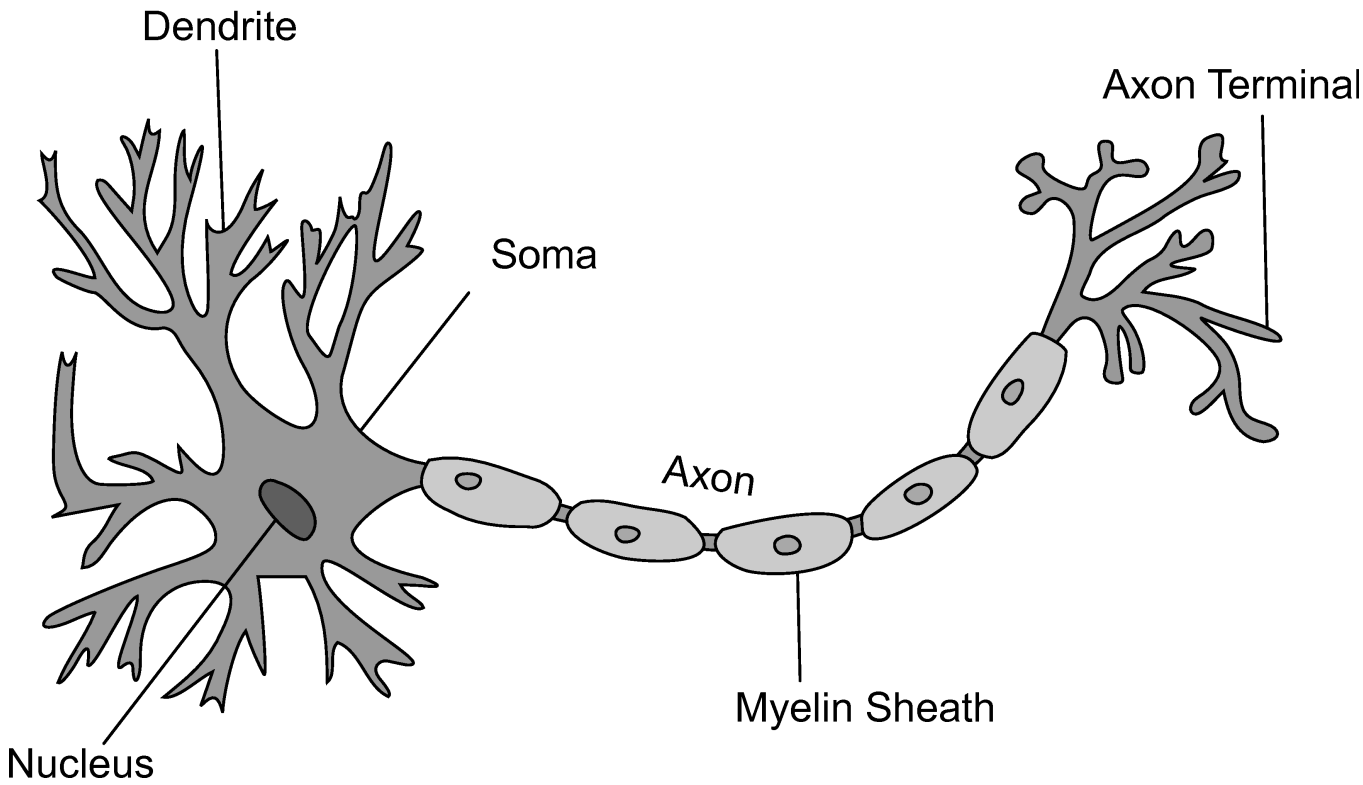


Vertebrate Eye



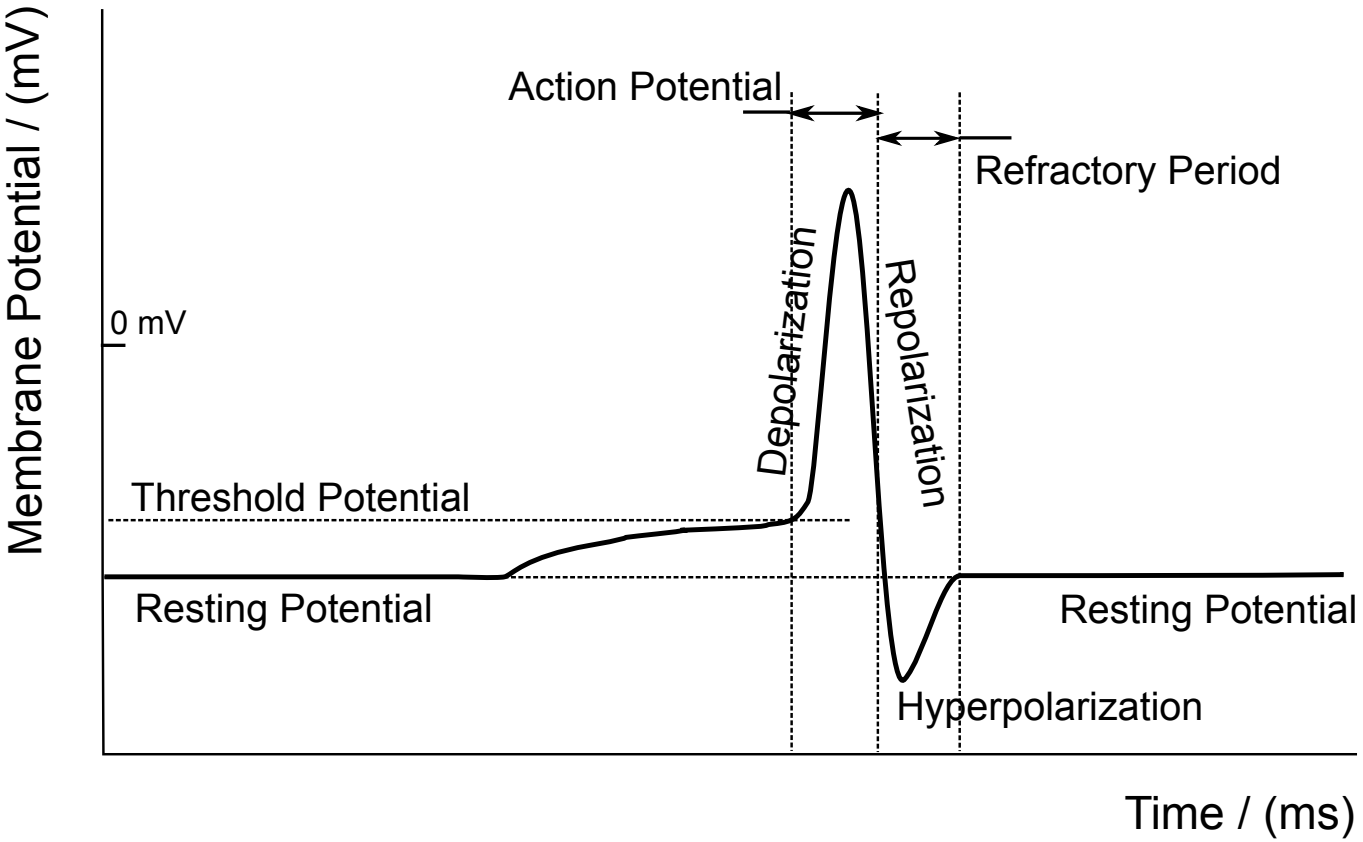
Schematic of the Vertebrate Retina

Building Blocks of Retina: *The Neurons*



Schematic of the a Neuron

Neural Information: *"Spikes"*



Action Potential

Modeling the Retina: *General Construction*

Many Possible Strategies

- Filter models (Eg. Rodieck, 1965, Citron & Marmarelis, 1987)
- Spiking models (Levine, 1992, Fohlmeister *et al.*, 1990)
- Stochastic models (Keat *et al.*, 2001)

Proposed Modeling Strategy

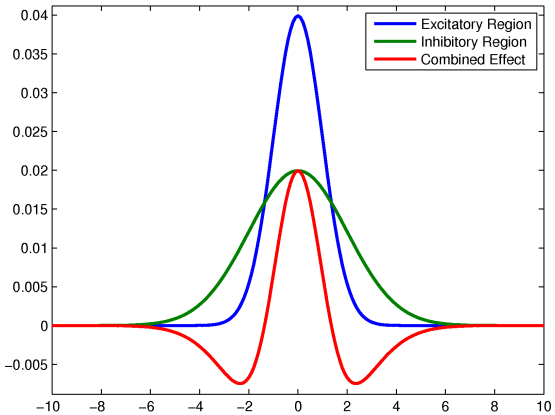
- Hybrid Model:
  - Upper layers of the retina are collectively modeled as a collection of filters, matching the known structural features
  - Individual ganglion cells are modeled with a spiking cell model
  - Upper layer (filter) responses synapse to the ganglion cells

Modeling the Retina: *Key Cellular Properties*

Receptor Fields: Retinal cells have different receptor field structures (Dearworth & Granda, 2002)

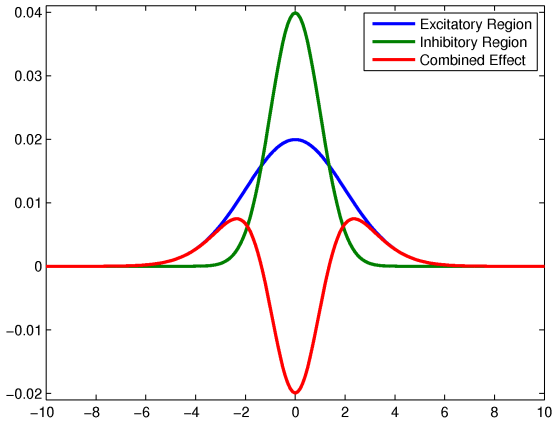
- ON Cells: Excitatory center & inhibitory surrounding
- OFF Cells: Inhibitory center & excitatory center
- ON/OFF Cells: Excitatory center, inhibitory rim and another outer excitatory rim

- ON Cells



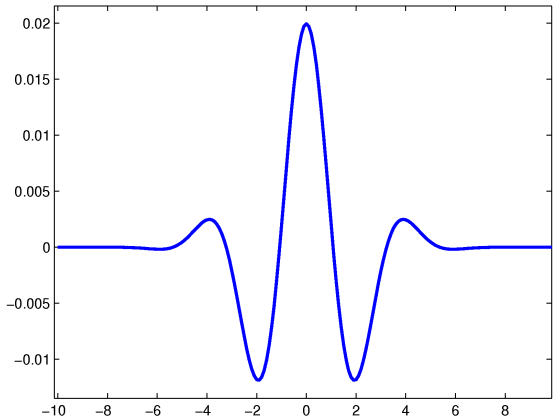
Difference of Gaussian Functions

- OFF Cells



Difference of Gaussian Functions

- ON/OFF Cells



Gabor Functions: Gaussian times cosine

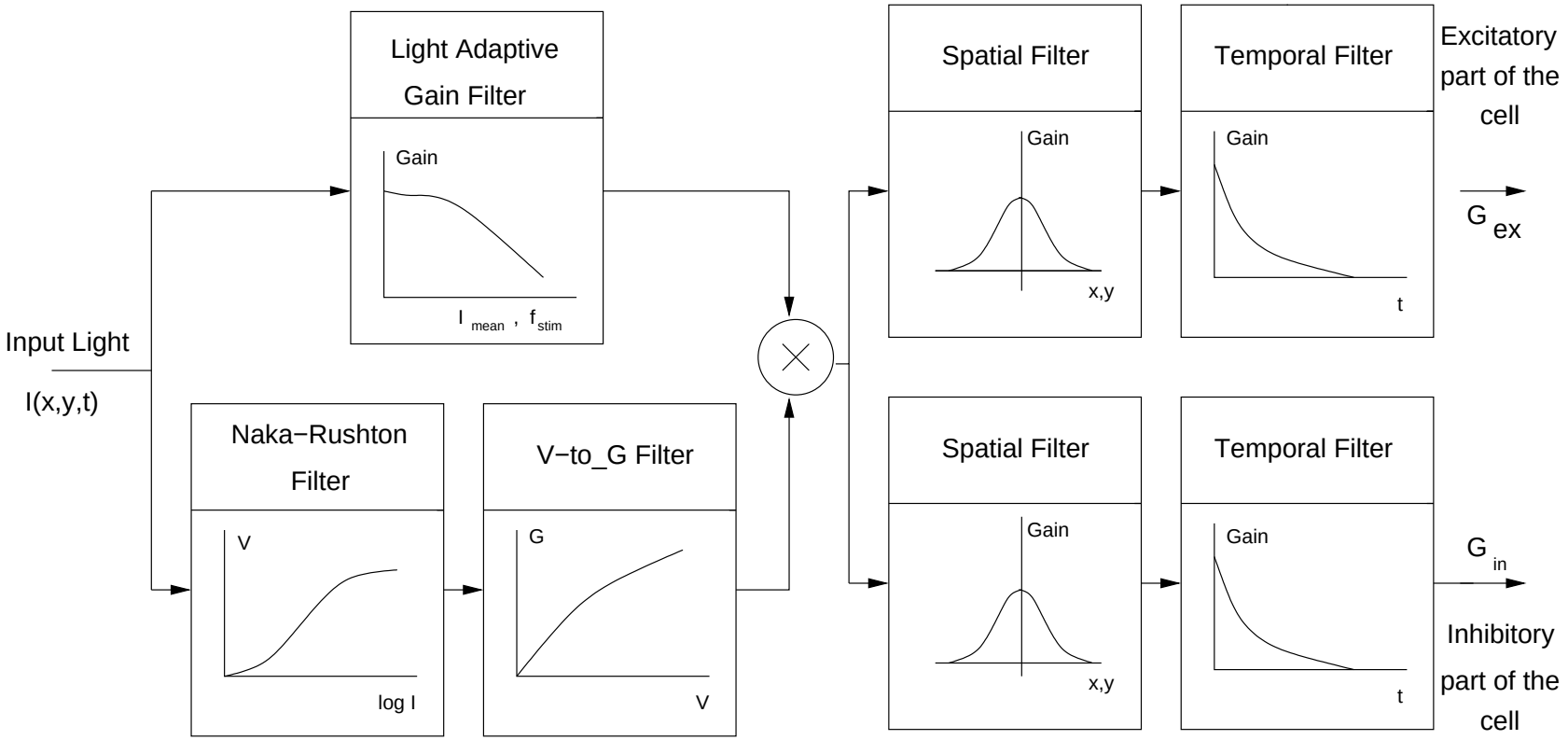
Direction Sensitivity: (Bowling, 1980, Rosenberg & Ariel, 1990)

- Some cells are sensitive to motion direction / optical flow
- Direction sensitive cells are mostly ON/OFF type
- They have a smaller cell body size
- Directional response is often characterized by Limaçon functions
- Has 3 main directional sensitivities: 180°, 40° and -75° (Bowling, 1980)

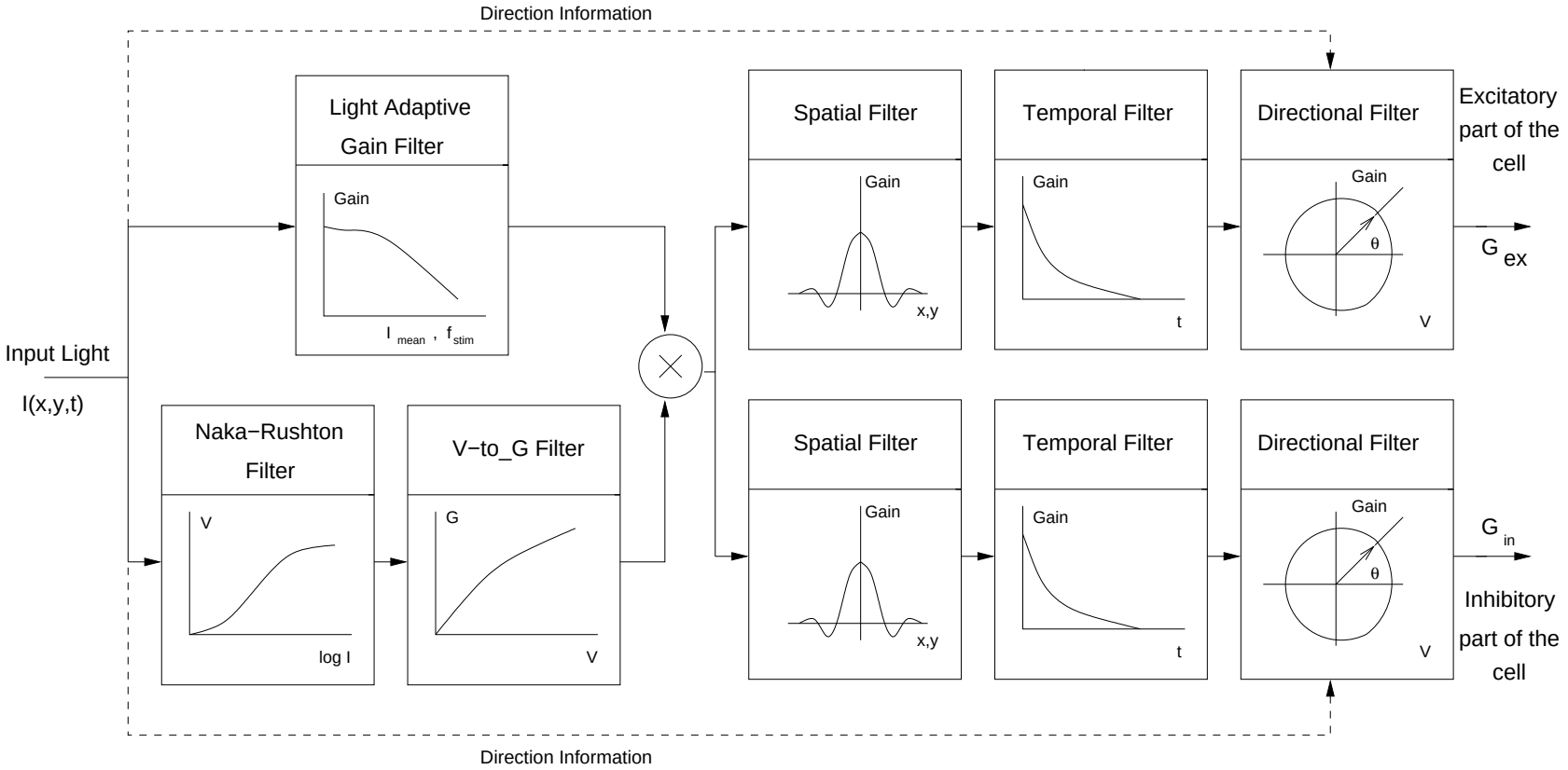
Classification of Cell Types:
(Marchiafava & Weiler, 1980, Dearworth & Granda, 2002,)

- Cells with larger cell body size are called A cells
  - Can be ON type or OFF type
  - Not directionally sensitive
- Cells with smaller cell body size are called B cells
  - ON/OFF type
  - Directionally sensitive

Modeling the Retina: *Block Diagrams – A Cell*



Modeling the Retina: *Block Diagrams – B Cell*



Modeling the Retina: *Common Filters*

Naka Rushton Filter: (Baylor & Hodgkin, 1973)

- Models the Phototransconduction events of the photoreceptors

$$R(I) = \frac{R_m I^n}{I^n + a^n}$$

Voltage-to-Conductance Filter: (Baylor & Hodgkin, 1973)

- Converts the voltage output from the NR filter to a conductance response

$$G(V) = g_{max} + \frac{1}{R_{in}} \cdot \frac{(E_{rest} - V - E'_{rest})}{(E_{rest} - V - E_{Na^+})}$$

Modeling the Retina: *Common Filters*

Light Adaptive Gain Filter: (Tranchina *et al.*, 1984)

- Models the adaption of cells to changing light conditions

$$H(f, I_0) = H_n \left[\frac{A(f)}{1 + I_0 B(f)} \right]$$

No significant effect for our problem as constant intensity input is used.

Temporal Filter: (Borg-Graham, 2001)

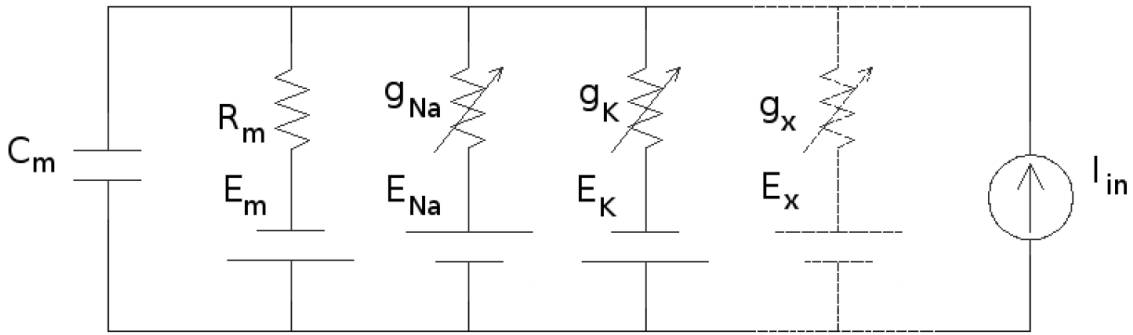
- Represent the temporal dynamics of the cellular processes

$$s(t) = a\, \mathrm{e}^{-t/\tau_a} + b\, \mathrm{e}^{-t/\tau_b}$$

Too fast for the speeds used. No significant effect again.

Modeling the Retina: *Spiking Cell Model*

Hodgkin-Huxley Model :



- Nobel winning work of Alan Hodgkin and Andrew Huxley in 1952.
- Mathematically describes the action potential generation with electrical characterization of ion channel dynamics of a cell.
- Ion channels are modeled as voltage dependent conductances and channel potentials.
- Can be generalized to accommodate various cellular ion channel dynamics as well as noise.

Modeling the Retina: *Hodgkin-Huxley Model*

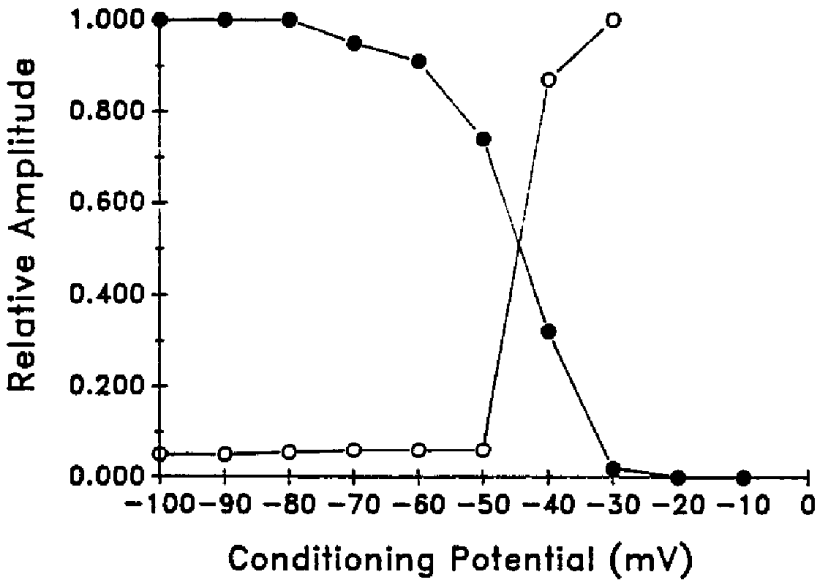
$$
\begin{aligned}
-C\frac{dV}{dt} &= I_{Na^+}(V,t) + I_{K^+}(V,t) + \cdots + I_{leak}(V,t) + I_{input}(V,t) \\
I_{input}(V,t) &= g_{exc}V + g_{inh}(V - V_{inh}); \qquad V_{inh} = -0.07\text{mV}
\end{aligned}
$$

For any channel current $I_x(V,t)$, with channel opening parameter r and channel closing parameter s,

$$
\begin{aligned}
I_x(V,t) &= \bar{g}_x r^m s^n (V - V_0) \\
\frac{dr}{dt} &= \alpha_r(1-r) - \beta_r r \\
\frac{ds}{dt} &= \alpha_s(1-s) - \beta_s s
\end{aligned}
$$

The α and β quantities can be one of three choices

$$
\alpha_\cdot, \beta_\cdot = \begin{cases} \dfrac{A(V-V_0)}{\exp(-(V-V_0)/B) - 1} & \textit{(Linoid)} \\ \dfrac{A}{\exp(-(V-V_0)/B) + 1} & \textit{(Sigmoid)} \\ A\exp(-(V-V_0)/B) & \textit{(Exponential)} \end{cases}
$$



Steady state voltage-current graph for the sodium channel. (From Lasater & Witkovsky, 1990

When, for example, $\frac{dr}{dt} = 0$

$$r_{ss} = \frac{\alpha_r}{\alpha_r + \beta_r}$$

Need to find two functional forms and the parameter values for α_r, β_r.

Modeling the Retina: *Ion Channel Parameters & Noise*

Parameter Estimation for HH Model:

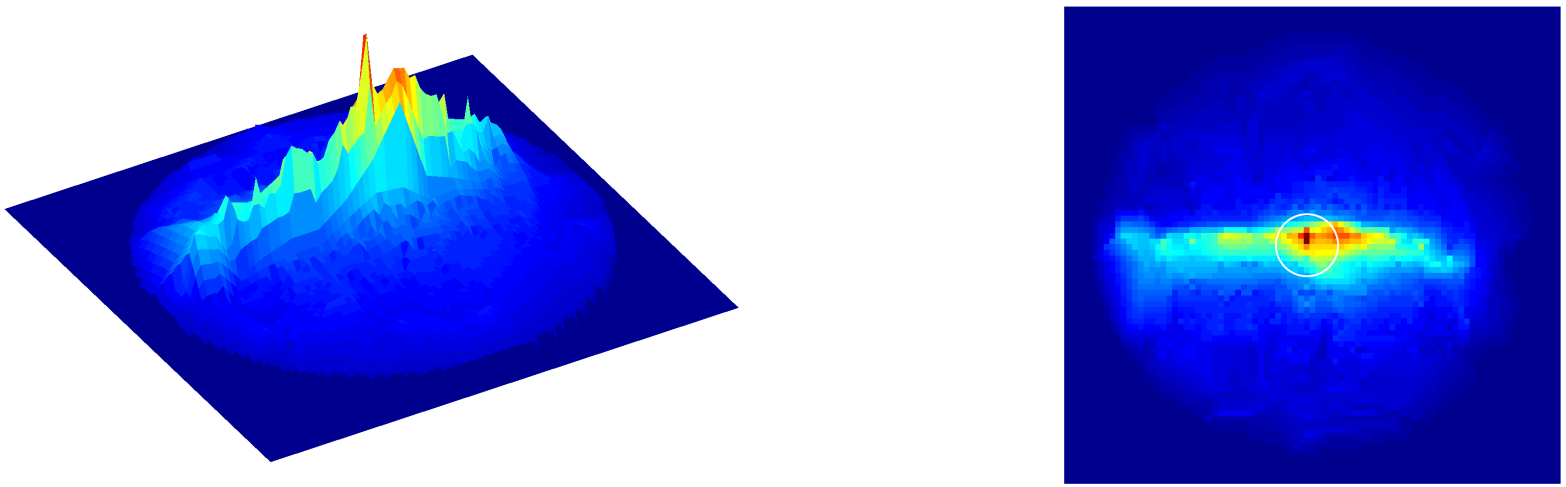
- Lasater & Witkovsky, 1990, and Liu & Lasater, 1994, reported the electrical properties of turtle ganglion cells
- Hodgkin-Huxley Model requires parameters to be estimated for a set of nonlinear functions using only steady state observations
- Adaptations of gradient search and steepest descent methods were used for parameter estimation
- Was exceedingly difficult to match the spike responses due to high dimensionality of parameters

Noise:

- Retinal cells produce a lot of activity due to background noise
- Noise is modeled as a random current input to the HH Model

Cell Distribution on the Turtle Retina: *The Visual Streak*

- Cells of the turtle retina has a visual streak structure, as opposed to the fovea structure found in, for example in primates.
- Cell distributions were computed using the data reported by Peterson & Ulinski, in 1979 and 1982.
- A patch from the center of the retina is used in our study.



Distribution of cells on turtle retina; color coded according to cell density

Problem Description

Main Problem :

1. How are the motion parameters of a target moving along the patch encoded by the cells?
2. How well can we decode the motion parameters given the patch response?

Classes of Motion Parameters :

1. Direction of motion of a target moving at constant speed, across the center of the patch
2. Speed of a target moving at constant speed, across the center of the patch

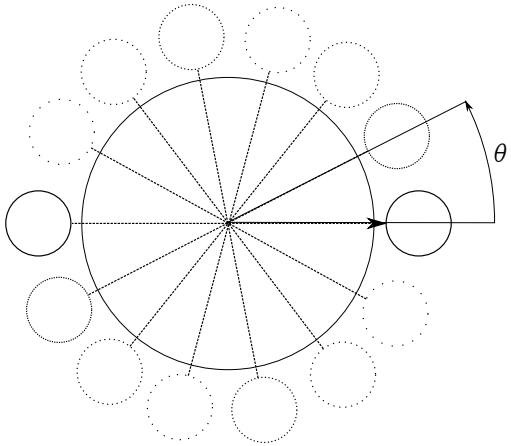
Eventual Goal :

- Can we recursively estimate the motion parameters (speed and direction) of a target given the cell responses?

Experiment Setup

Patch Properties

- Located at the center of the visual streak
- Total 520 cells:
  - A Cells $\approx$ 50 each ON & OFF
  - B Cells $\approx$ 140 from each type
- Simulation repeated 60 times for each input



Schematic depiction of the experimental setup

"Direction Experiment"

- Fixed speed (taked 0.8 seconds to cross the patch)
- Angles 0° to 358° at 2° steps
- Left to right along the visual streak is 0°

"Velocity Experiment"

- Multiple speeds takes 0.4 to 2 seconds to cross the patch
- Each with Angles 0° to 330° at 30° steps

Analyzing Neural Data

Challenge: Large dimension of data.

Principle Component Analysis PCA

- Also known as discrete Karhunen-Loève transform (KLT)
- Used to obtain a lower dimensional representation of recorded data.
- Popular tool in neural information processing

Point Processes

- Counting processes, in particular Poisson Processes, are intensively been adopted for many neurological problems (E.g. Brown, Barbieri, Snyder)
- Neural activities caused by action potentials constitute point processes

Counting Processes

Definition (Counting Processes - Snyder 1971)

Let $\omega \in \Omega$ be a finitely denumerable point set in a closed interval $[0, T]$ where $T > 0$. Therefore, each ω can be enumerated in the form $\{t_1(\omega), t_2(\omega), \ldots, t_n(\omega)\}$, where $0 \leq t_1(\omega) \leq t_2(\omega) \leq \cdots \leq t_n(\omega) \leq T$ and $n < \infty$. Define $\{N_t(\omega) : 0 \leq t \leq T\}$ by

$$N_t(\omega) = \begin{cases} 0, & 0 \leq t \leq t_1(\omega) \\ 1, & t_1(\omega) < t \leq t_2(\omega) \\ \vdots & \\ n-1, & t_{n-1}(\omega) < t \leq t_n(\omega) \\ n, & t_n(\omega) < t \leq T \end{cases}$$

Then for each fixed $t \in [0, T]$, $N_t(\omega)$ is an integer-valued, discrete random variable. For each fixed $\omega \in \Omega$, $N_0(\omega) = 0$ and $N_t(\omega)$ is a piecewise constant, left continuous function of t with unit positive jumps at $t_1(\omega), t_2(\omega), \ldots, t_n(\omega)$. Then $\{N_t(\omega) : 0 \leq t \leq T\}$ is a counting process.

Poisson Processes

Definition (Poisson Process)

A Poisson process is a is a counting process $\{N_t : t \geq t_0\}$ with the following three properties:

1. $Pr[N_0 = 0] = 1$
2. For $t_0 \leq s < t$, the increment $N_{s,t} = N_t - N_s$ is Poisson distributed with parameter $\Lambda_t - \Lambda_s$. That is,

$$Pr\left(N_{s,t} = n\right) = \frac{\left(\Lambda_t - \Lambda_s\right)^n}{n!} exp\left(-\left(\Lambda_t - \Lambda_s\right)\right),$$

   for $n = 0, 1, 2, \ldots$ and Λ_t is a nonnegative, nondecreasing function of t.
3. $\{N_t : t \geq t_0\}$ has independent increments.

Useful Definitions

Definition (Orderliness)

A counting process $\{N_t : t \geq t_0\}$, and the underlying point process as well, is called orderly at time $t \geq t_0$ if for any given $\epsilon > 0$, there exists a $\delta > 0$, only dependent on t and ϵ such that,
$Pr\left(N_{t,t+\delta'} > 1\right) \leq \epsilon Pr\left(N_{t,t+\delta'} = 1\right)$ for all $\delta' \in (0, \delta)$.

Definition (Conditional Orderliness)

Let P be an arbitrary event determined by the random variables $\{N_\sigma : t_0 \leq \sigma < t\}$. A counting process $\{N_t : t \geq t_0\}$, and also the underlying point process, is conditionally orderly at time $t \geq t_0$ if for any P and any $\epsilon > 0$, there exists a $\delta > 0$, which depends only on t and ϵ such that $Pr\left(N_{t,t+\delta'} > 1 \,|P\right) \leq \epsilon Pr\left(N_{t,t+\delta'} = 1 \,|P\right)$ for all $\delta' \in (0, \delta)$.

Definition (Evolution Without Aftereffects)

A point process on $[t_0, \infty)$ is said to evolve without aftereffects if for any $t \geq t_0$, the realization of points during the interval $[t, \infty)$ does not depend in on the of events that have transpired in the interval $[t_0, t)$.

Conditions for Poisson Processes

Theorem (Conditions for Poisson Processes)

Let $\{N_t : t \geq t_0\}$ be the counting process associated with a point process on $[t_0, \infty)$. Suppose that:

1. *The point process is uniformly orderly on $[t_0, t)$ for all $t \geq t_0$,*
2. *The point process evolves without aftereffects,*
3. *Points does not occur at preassigned times,*
4. *There is no finite interval in $[t_0, \infty)$ where points occur for certainty,*
5. *$Pr(N_{t_0} = 0) = 1$*

Then $\{N_t : t \geq t_0\}$ is a Poisson counting process with a continuous parameter function.

Conditions for Poisson Processes

Theorem (Alternate Conditions for Poisson Processes)

Let $\{N_t : t \geq t_0\}$ be the counting process associated with a point process on $[t_0, \infty)$. Suppose that:

1. *The point process is conditionally orderly,*
2. *For all $t \geq t_0$ and for an arbitrary event P associated with the random variables $\{N_t : t_0 \leq \sigma < t\}$, the limit of $Pr\left(N_{t,t+\delta} = 1 \,|P\right)/\delta$ exists as δ tends to zero, and the limit is finite, integrable function of t alone. Let this limit function be λ_t, $\lambda_t = \lim_{\delta \downarrow 0} \dfrac{Pr\left(N_{t,t+\delta} = 1 \,|P\right)}{\delta}$ and $\int_s^t \lambda_\sigma \, d\sigma$ exists and is finite for all finite intervals $[s, t)$, $t_0 \leq s \leq t$,*
3. *$Pr\left(N_{t_0} = 0\right) = 1$.*

Then $\{N_t : t \geq t_0\}$ is a Poisson counting process with an absolutely continuous parameter function $\Lambda_t = \int_{t_0}^t \lambda_\sigma \, d\sigma$

Applying Poisson Processes to Model Neuron Data !?

- When a cell fires, there is a brief refractory period during which the cell can no longer produce spikes.
- This violates the condition "Evolution without aftereffects".
- Furthermore, the actual spiking pattern depends on the input light condition, and this complicates the process.
- So, neural signals cannot be modeled using the simple Poisson processes!

Self-Exciting Counting Processes

Definition (Self-Exciting Counting Processes)

Let $\{N_t : t \geq t_0\}$ be a counting process and assume the following properties:

1. $\{N_t : t \geq t_0\}$ is conditionally orderly
2. The limit of the function $a(\Delta t, N_t)$ defined by

$$a(\Delta t, N_t) = \begin{cases} Pr\left(N_{t,t+\Delta t} = 1 \,|N_t\right)/\Delta t, & \text{for } N_t = 0 \\ Pr\left(N_{t,t+\Delta t} = 1 \,|N_t; \tau_1, \tau_2, \ldots, \tau_{N_t}\right)/\Delta t, & \text{for } N_t \geq 1 \end{cases}$$

   exists and finite as Δt tends to zero for almost every realization of $\{N_t : t \geq t_0\}$. Here τ_i is the time of occurrence of the i^{th} count of the point process.

3. $Pr(N_0 = 0) = 1$

Then $\{N_t : t \geq t_0\}$ is a self-exciting counting process.

Doubly Stochastic Poisson Process

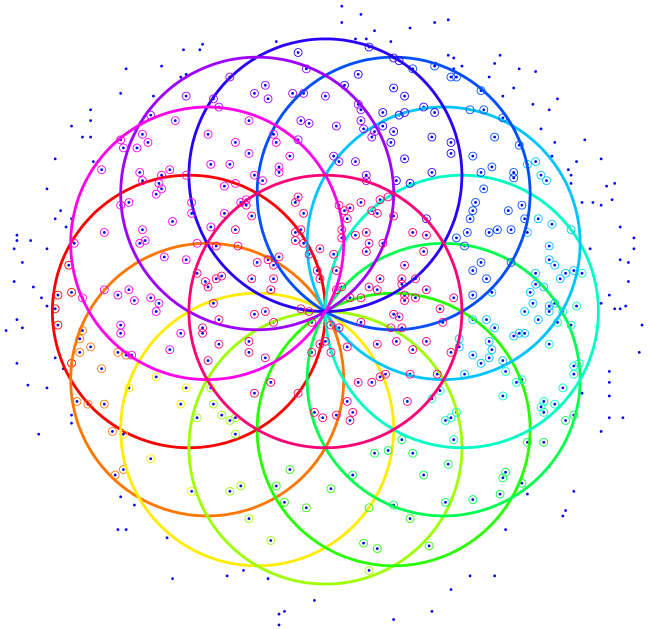
Definition (Doubly Stochastic Poisson Process)

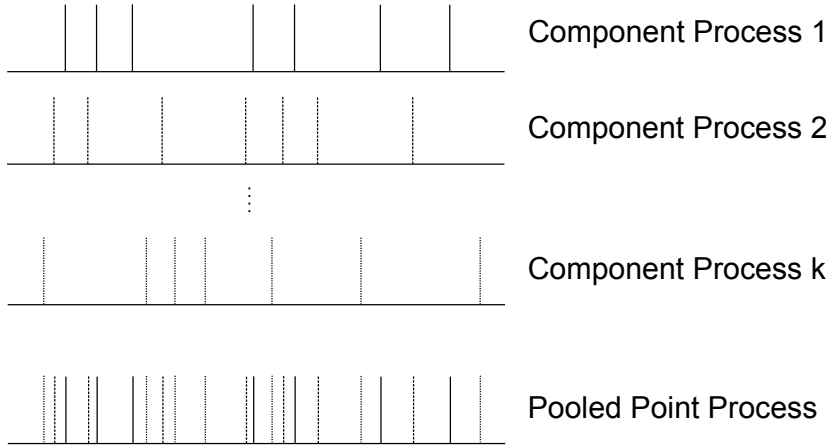
$\{N_i(t) : t \geq t_0\}$ is a doubly stochastic Poisson process with intensity process $\{\lambda_t(x_t) : t \geq t_0\}$ if for almost every given path of the process $\{x_t : t \geq t_0\}$, N is a Poisson process with intensity function $\lambda_t(x_t)$. In other words, $\{N_i(t) : t \geq t_0\}$ is conditionally a Poisson process with intensity function $\lambda_t(x_t)$ given $\{x_t : t \geq t_0\}$.

Theorem (Characterization of a Doubly Stochastic Poisson Process as a Self-Exciting Point Processes)

Let $\{N_t : t \geq t_0\}$ be a doubly stochastic Poisson process with intensity process $\{\lambda_t(x_t) : t \geq t_0\}$, and suppose $E[\lambda_t(x_t)] < \infty$ for all $t \geq t_0$. Then $\{N_t : t \geq t_0\}$ is a self-exciting counting process with intensity process $\left\{\hat{\lambda}_t : t \geq t_0\right\}$, where $\hat{\lambda}_t = E[\lambda_t(x_t) | N_\sigma : t_0 \leq \sigma < t]$.

Pooled Counting Processes





Theorem (Limit theorem for Pooled Point Processes)

Suppose the component counting processes $\{N_i(t) : t \geq t_0\}$ *for* $i = 1, 2, \ldots, k$ *are mutually independent and uniformly sparse for time* t *finite. Then the pooled counting process* $\{M_k(t) : t \geq t_0\}$ *converges in distribution to a Poisson process with intensity* $\{\lambda_t : t \geq t_0\}$ *if and only if both* $\lim_{k\to\infty} \sum_{i=1}^{k} Pr\left(N_i(t) > 1\right) = 0$ *and* $\lim_{k\to\infty} \sum_{i=1}^{k} Pr\left(N_i(t) = 1\right) = \int_{t_0}^{t} \lambda_\sigma d\sigma$ *for* $t_0 \leq t < \infty$

Applying the Theory of Counting Processes

- We can model the retinal responses as self exciting Point Processes
- Since individual retinal cells are independent encoders and therefore, also uniformly sparse, we can pool the responses of a "large" number of cells and approximate by an inhomogeneous Poisson Process.
- Since this response is driven by an input which is, in general stochastic, we can expect the approximate limit process to be a Doubly Stochastic Poisson Process.
- We use some more theorems for this verification purpose.

Time Rescaling Theorem & Interarrival Times

Theorem (Time Rescaling Theorem)

Let $0 < u_1 < \cdots < u_n < T$ be a realization from a point process with conditional intensity function $\lambda(t)$, satisfying $0 < \lambda(t)$ for all $t \in (0, T]$. Define the transformation

$$\Lambda(u_k) = \int_0^{u_k} \lambda(u)\, du,$$

for $k = 1, \ldots, n$, and if $\Lambda(t) < \infty$ almost surely for $t \in (0, T]$, then the sequence $\Lambda(u_1), \ldots, \Lambda(u_n)$ are a Homogeneous Poisson process with unit rate.

Theorem (Interarrival Times of Homogeneous Poisson Processes)

For a homogeneous Poisson counting process with intensity λ, the interarrival times (the time interval between two occurrences of the underlying point process) $t_1, t_2, \ldots, t_n$ are independent and identically distributed with the common distribution being exponential with parameter λ.

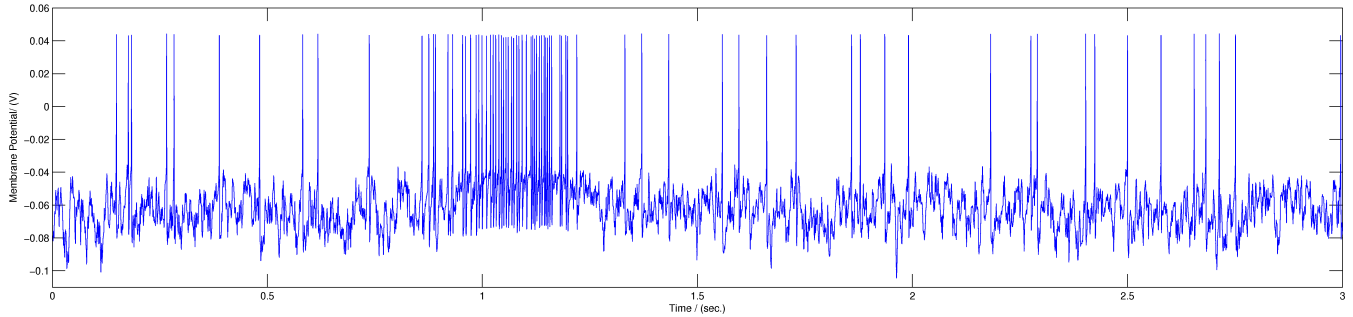
Encoding : *Estimating Intensity Function*

Four Step Process of Estimating $\Lambda_\theta(t) = \int_0^t \lambda_\theta(\sigma)\mathrm{d}\sigma$

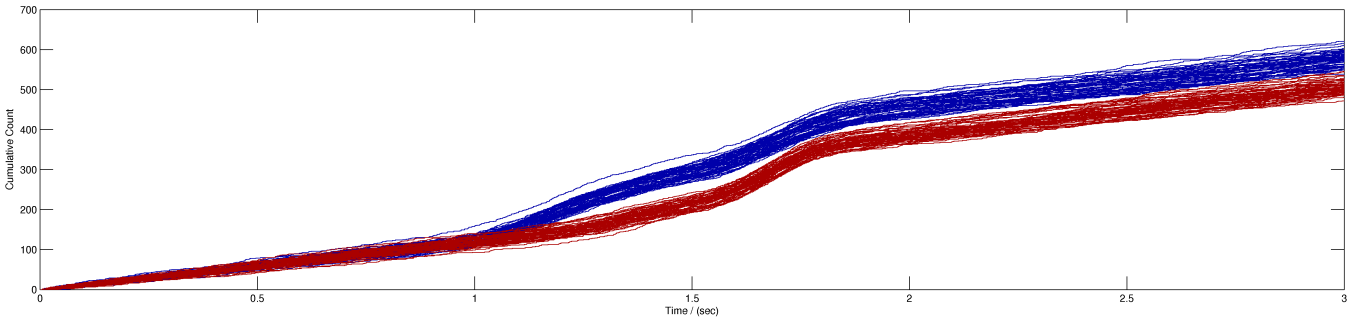
1. Find the spike time for the constituent cells of the pooled set
2. Find the N_t, the cumulative sum of spikes from the starting time ($t = 0$) up until time t
3. Find the mean, $E[N_t]$, over all the simulations for a given input θ
4. Smooth-out $E[N_t]$ using a smoothing spline or any other kernel smoothing method.

- This approach is efficient than the usual binning and spike rate calculation for estimating the intensity function $\lambda_\theta(t)$.
- We can use the spike rescaling theorem, interarrival time property along with the Kolmogorov–Smirnov test to verify indeed that the Poisson process model is valid.

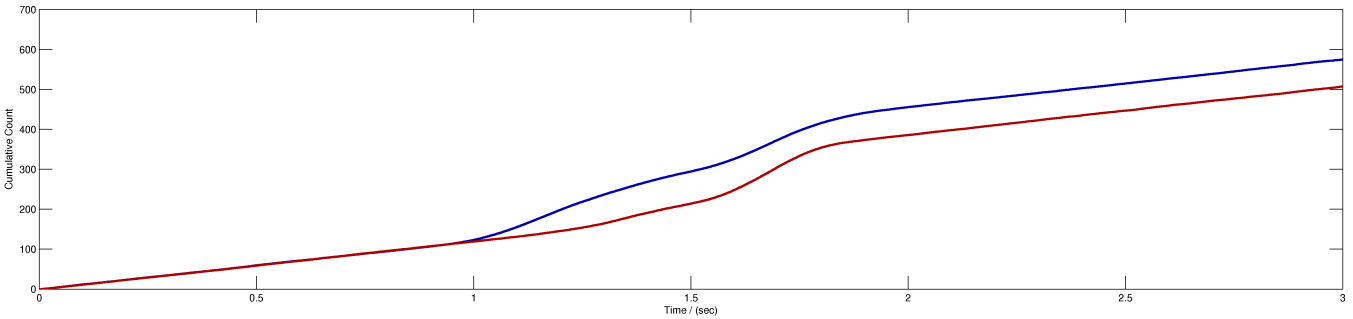
Sample Intensity Functions: An Example



Spike pattern of one cell



Some sample paths with 10 cells pooled together

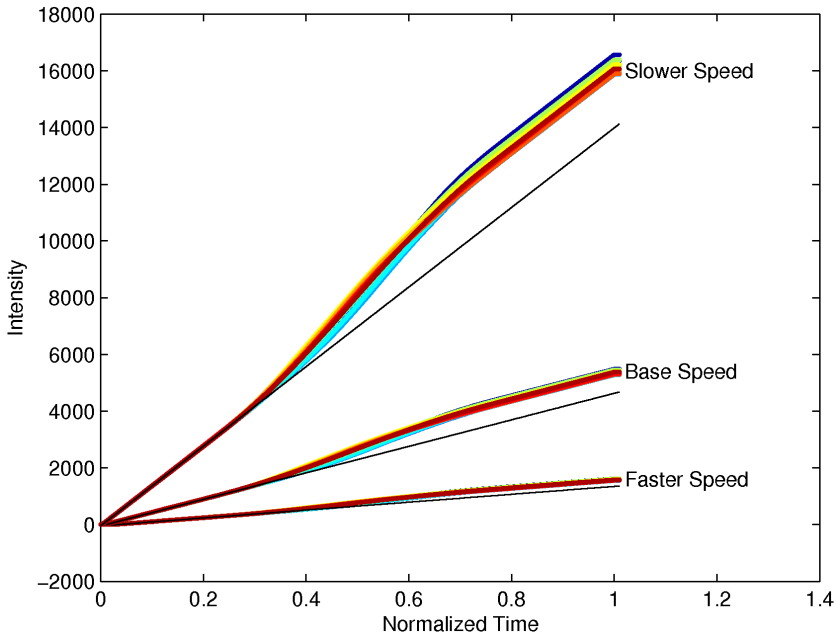


Mean activity pattern of the activity patterns above

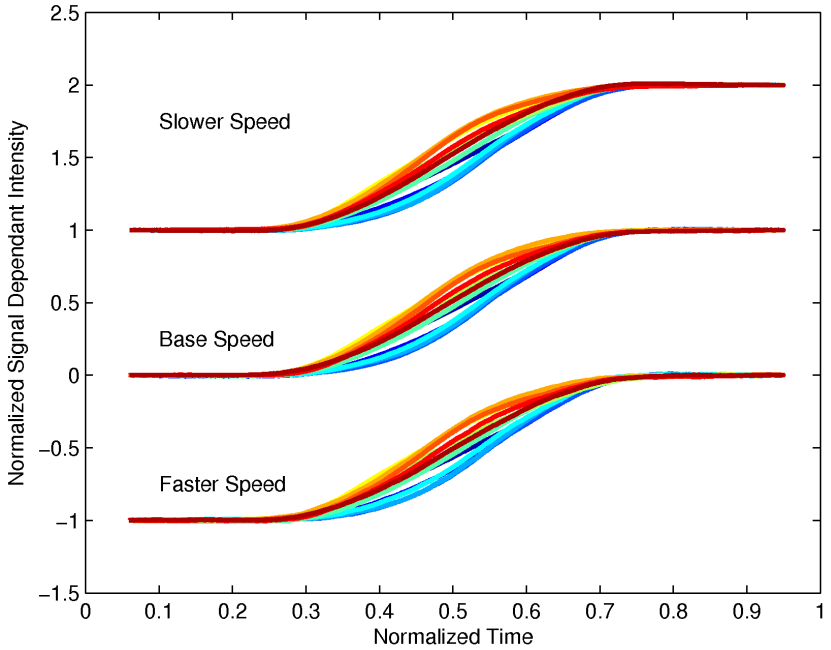
Intensity Functions: *Comparing Speeds*

- For comparing speeds, we need to assume a structure for the intensity process:
$$\lambda_{\theta,s}(t) = \lambda_0 + \lambda(t)$$
- The constant part λ_0 models the background activity.(Iyengar & Liao, 1997)
- λ_0 can be estimated using the portion of response before and after the input is incident on the patch
- The time dependent part $\lambda(t)$ models the input dependent component
$$\Lambda(t) = \int_0^t \lambda(\sigma)\mathrm{d}\sigma = \int_0^t \lambda_{\theta,s}(\sigma)\mathrm{d}\sigma - \lambda_0 t$$
- $\Lambda(t)$ functions are rescaled in time and normalized in amplitude

Intensity Functions: *Comparing Speeds*

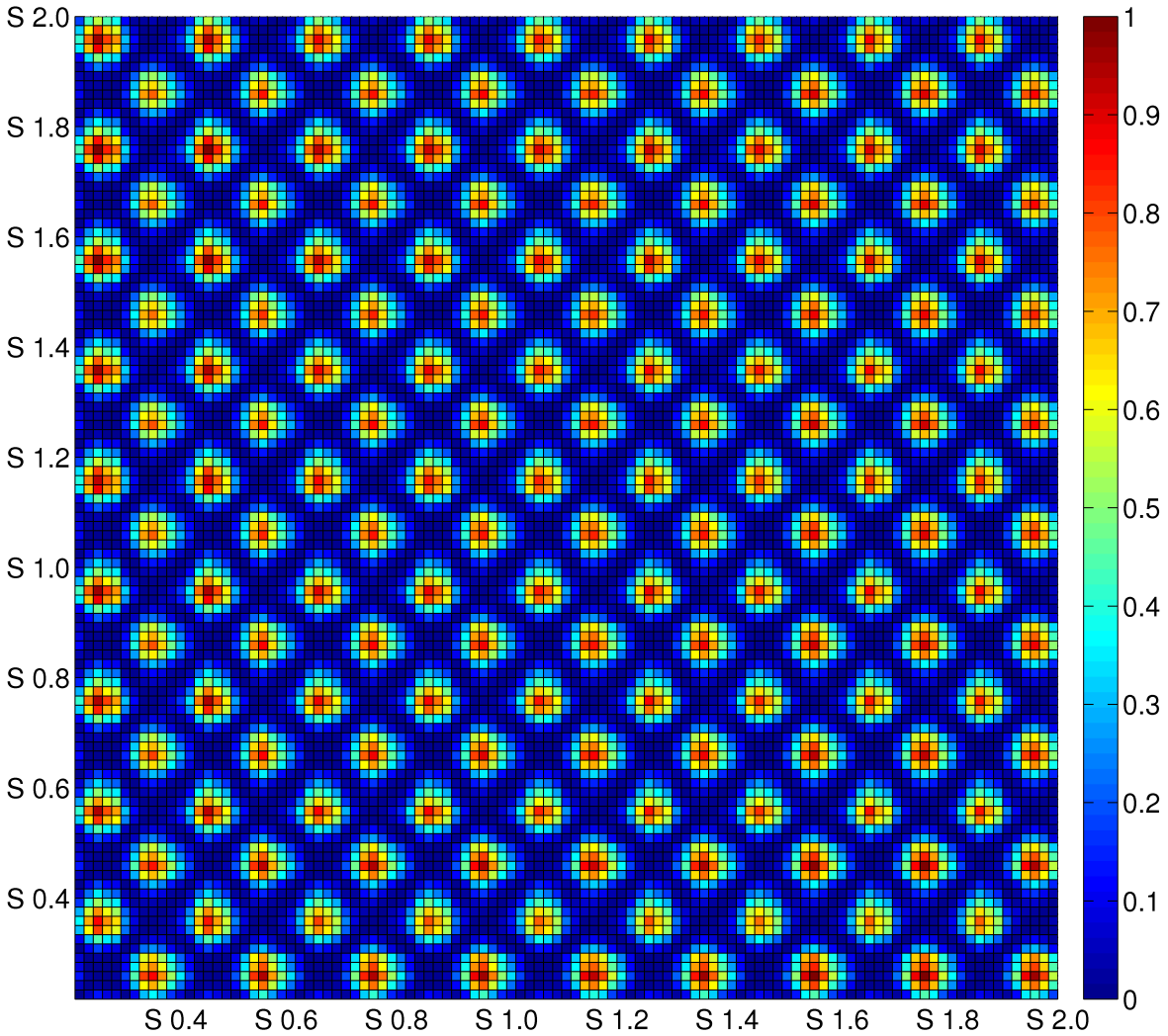


Intensity function and background activity (normalized)



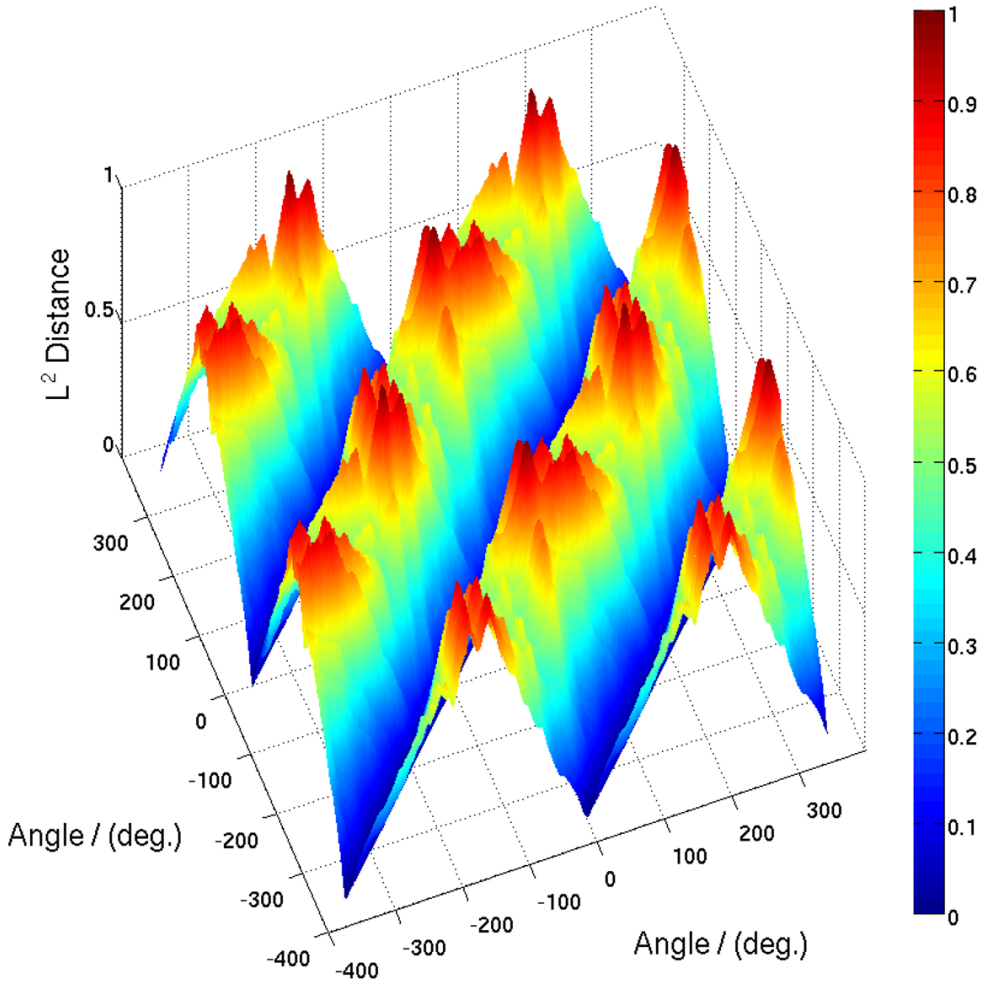
Signal dependent intensity (normalized)

Results: *Comparing Speeds*



The normalized L^2 distance between the intensity functions after linearly rescaling the time to a standard $[0, 1]$ time interval

Intensity Functions: Direction Experiment



The responses obtained for Angles 0° to 358° are repeated over from -360° to -2°, so that this graph compares the distance between the intensity functions of angles -360° to 358°.

Likelihood function and Hypothesis Testing

- The intensity functions suggest that the intensity functions are "considerably" different

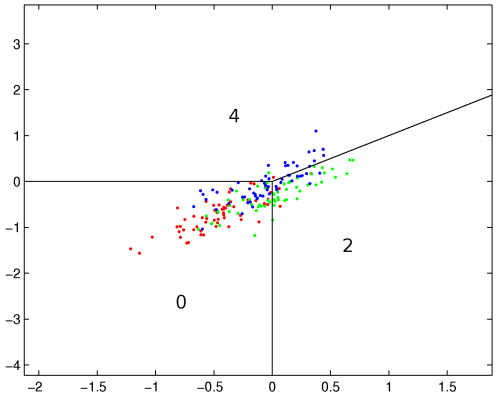
Likelihood Function for a Observed Poisson Process using Binning

- Suppose the observation interval $[0, T)$ is partitioned in to a set of k disjoint intervals (called bins), $[0, t_1)$, $[t_1, t_2)$, $\ldots$, $[t_{k-1}, T)$.
- Let n_i be the number of spikes observed during the i^{th} subinterval.
- Then the log-likelihood function can be easily found to be

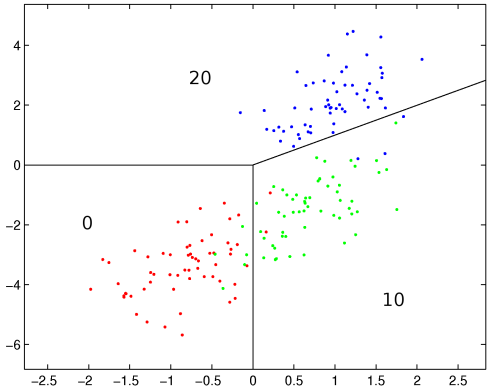
$$l_\theta(R) = -\int_{t_0}^{T} \lambda_\theta(\sigma)\mathrm{d}\sigma + \sum_{i=1}^{k} n_i \ln\left(\int_{t_{i-1}}^{t_i} \lambda_\theta(\sigma)\mathrm{d}\sigma\right)$$

- This can be applied for the hypothesis testing problem.

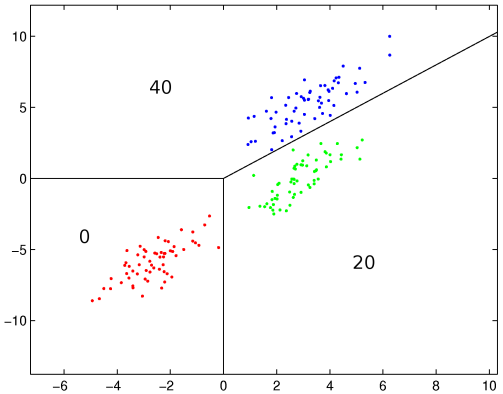
Decision Space



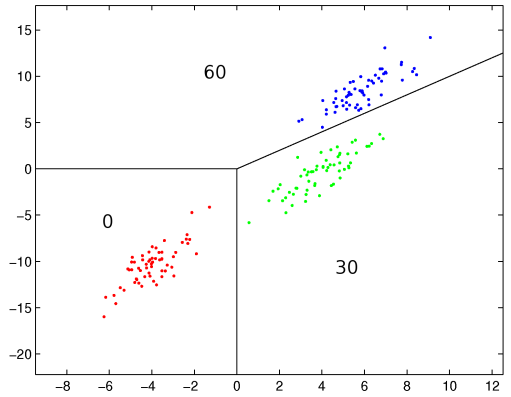
All cells 2° separation



All cells 10° separation



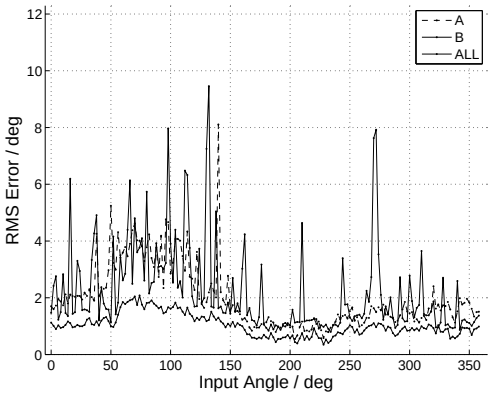
All cells 20° separation



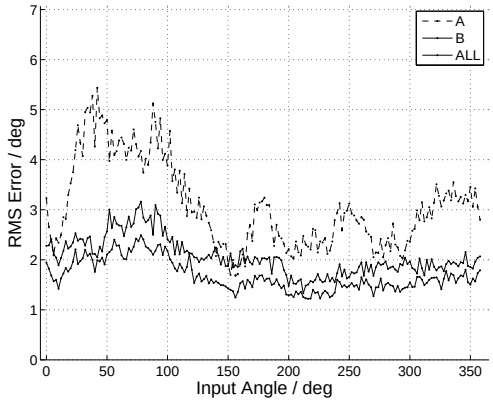
All cells 30° separation

Results: *Estimation Error in the Direction Experiment*

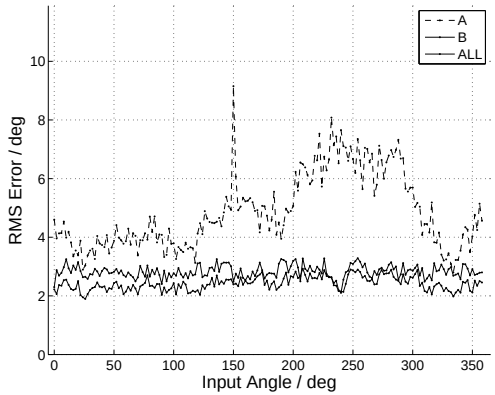
- Using 100 ms Estimation Window (Total simulation time 800 ms)



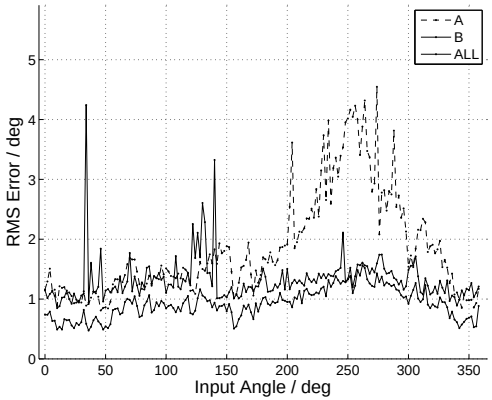
Starting at 200 ms



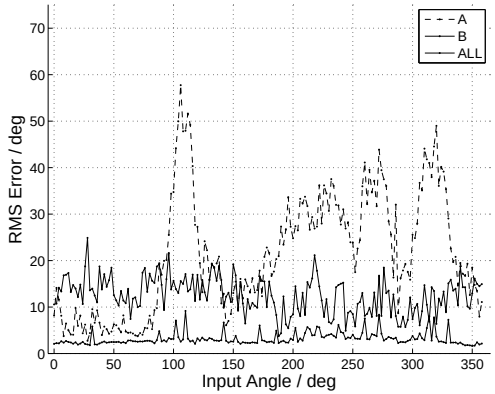
Starting at 300 ms



Starting at 400 ms



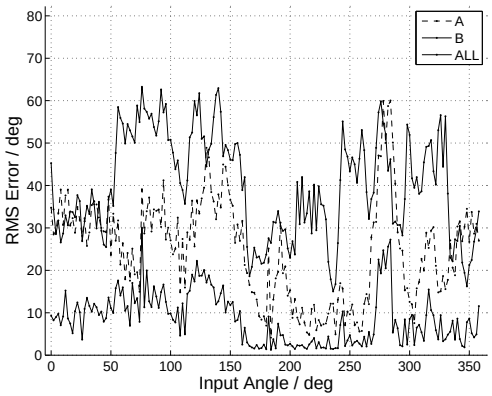
Starting at 500 ms



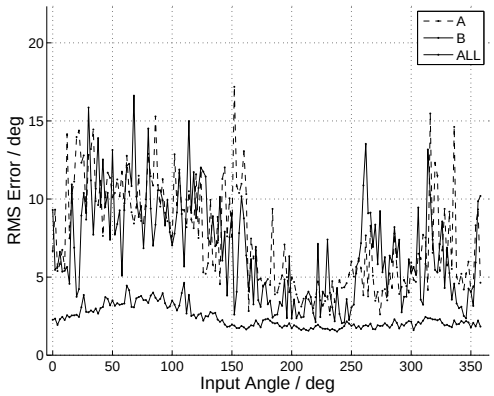
Starting at 600 ms

Results: *Estimation Error in the Direction Experiment*

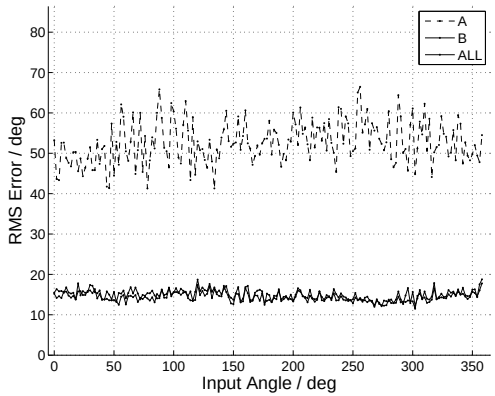
- Using 20 ms Estimation Window (Total simulation time 800 ms)



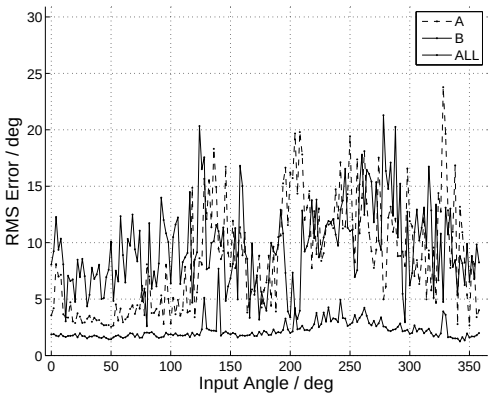
Starting at 200 ms



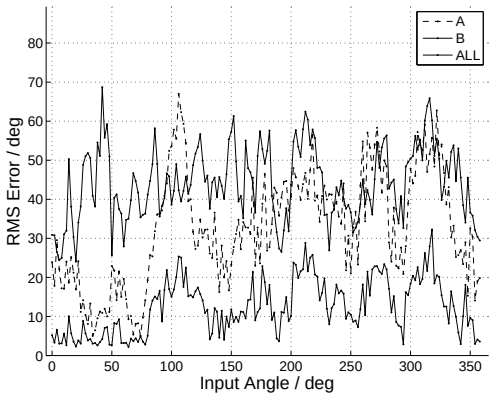
Starting at 300 ms



Starting at 400 ms



Starting at 500 ms



Starting at 600 ms

Results Summary & Conclusions

1. With only 3 principal directional sensitivities, the directional sensitive B-Cells of the retina can encode full 360° of motions.
2. Directional sensitive cells can estimate direction information using a very short time interval compared to the non-directional sensitive cells.
3. When the time interval is increased, non-direction sensitive cells improve in their detectability, but this is limited by the fact that their population is less than direction sensitive cells.
4. Retinal responses at different speeds cam be modeled as time and amplitude rescaling of the time varying part of the response

Future Work

- The results obtained here clearly demonstrates the motion encoding properties of the retina
- Goal is to develop a filter design which can recursively estimate the trajectory of a light input on the retina
- Possible strategies include nonlinear Kalman filters, estimation methods for doubly stochastic Poisson processes, etc.

THANK YOU

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