Tracking and Optimal Control Problems in Human Head/Eye Coordination

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Introduction

- The head/eye movement is a dynamics problem inspired by biology. It has been studied by scientists since the 19th century.
- Amongst them are prominent scientists like F. Donders, J. Listing, and H. von Helmholtz.
- Donders and Listing have proposed laws that govern the head and eye movements.
- Some of the recent studies on the subject include that of D. Robinson M. Ceylan, D. Tweed, J. Crawford, B. Glenn and T. Vilies.

Head/Eye Coordination Problem

- Head/eye coordination problem studies the case where the head and the eye act in coordination to capture and stabilize the image on the retina.
- In this scenario, the eye moves faster to capture the target and the head follows in a slower motion.
- Once the eye captures the target, the vestibuloocular reflex comes into action and the eye compensates for the head movement keeping the image on the retina stable (gaze is held constant with respect to the torso).
- In this paper we treat this as a tracking control problem of the eye where the head trajectory is known to the eye controller. The initial and the final conditions of the head are assumed to be known.

Approach

- We take a classical mechanics approach to address the problem.
- The head and the eye are modeled as spheres rotating about their centers.
- We study the dynamics in a constrained rotation space.
- The results are compared with measured human head/eye data.

Laws Governing the Possible Head/Eye Orientations

- The possible orientations that the head/eye can attain are governed by laws expressed in terms of a rotation vector.

- In the context of this research, we define the rotation vector as

$$\vec{r} = \tan\frac{\phi}{2}\,\vec{n}$$

where ϕ is the angle of counter-clockwise rotation about the unit vector $\vec{n}$.

Let $\vec{r} = (r_1, r_2, r_3)$.

Donders' Law

Donders' law states that starting from a primary eye/head position, any other eye/head orientation is obtained by a rotation vector constrained to lie on a two dimensional surface, called the Donders' surface.

The quadratic Donders' surface: $r_3 = a_0 + a_1 r_1 + a_2 r_2 + a_3 r_1 r_2 + a_4 r_1^2 + a_5 r_2^2$
Fick surface (Pan and tilt): $r_3 = -r_1 r_2$

Listing's Law

Listing's law states that the Donders' surface for the eye orientations is a plane perpendicular to the primary gaze direction which we refer to as the Listing's plane.

Listing's plane: $r_3 = 0$

Parameterizing the Rotations

- In order to model the rotational dynamics, we need a parameterization for the space of rotations ($SO(3)$).
- The elements of the rotation group $SO(3)$ are 3×3 matrices.
- One parameterization of $SO(3)$ would be the Euler angles.
- We skip the details of the parameterizations used in this paper.

Deriving the Equations of Motion

- We use the Euler-Lagrange equations to derive the equations of motion for the system.
- This requires the expressions for the kinetic and the potential energies of the system.
- Let (x_1, x_2, x_3) be a parameterization of $SO(3)$.
- Let $\dot{X} = (\dot{x_1}, \dot{x_2}, \dot{x_3})^T$. Then the kinetic energy KE of the system can be written in the form

$$KE = \frac{1}{2}\,\dot{X}^T G\,\dot{X}.$$

where G is the Riemannian metric.

Equations of Motion on $SO(3)$

- If the potential energy of the system is represented by V, the Lagrangian of the system can be written as

$$L = KE - V.$$

- The equations of motion of the system can be calculated by using the Euler-Lagrange equations expressed as

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\beta}} - \frac{\partial L}{\partial \beta} = \tau_\beta$$

  where β is a variable.

- The resulting equations of motion can be expressed as

$$G\ddot{X} + \dot{G}\dot{X} - \frac{1}{2}\dot{X}^T \nabla_X G\dot{X} + \nabla_X V = \Gamma$$

  where $\nabla_X = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right)$ and Γ is the torque vector.

- We write this as $\ddot{X} = F$.

Optimal/Tracking Control

- The optimal control problem minimizes the cost function

$$\int_0^1 \left[\frac{\alpha}{2}\Gamma^T\Gamma + \frac{\beta}{2}C^T C\right] dt$$

under the constraints $\ddot{X} = F$ and $D \equiv 0$ (Donders' Constraint).

- To achieve this, we apply calculus of variation to the function

$$J = \int_0^1 \left[\frac{\alpha}{2}\Gamma^T\Gamma + \frac{\beta}{2}C^T C + p^T\left(F - \ddot{X}\right) + \lambda D + \frac{1}{2}\dot{\lambda}^T\epsilon\dot{\lambda}\right] dt$$

where p and λ are Lagrange multipliers and $C = f - X$ where f is a trajectory to be tracked.

Optimal/Tracking Control Contd

- We define the Hamitonian to be

$$H = \frac{\alpha}{2}\Gamma^T\Gamma + \frac{\beta}{2}C^T C + p^T F + \lambda D + \frac{1}{2}\dot{\lambda}^T \epsilon \dot{\lambda}.$$

Then

$$J = \int_0^1 \left[H - p^T \ddot{X} \right] dt.$$

- By taking variations with respect to X, Γ, p, and λ, we obtain differential equations,

$$\begin{aligned} \ddot{X} &= \frac{\partial H}{\partial p} = F; \quad \ddot{p} = \frac{\partial H}{\partial X} - \frac{d}{dt}\left(\frac{\partial H}{\partial \dot{X}}\right) \\ \ddot{\lambda} &= \epsilon^{-1}\frac{\partial H}{\partial \lambda}; \quad \Gamma = -\frac{1}{\alpha} G^{-1} p. \end{aligned}$$

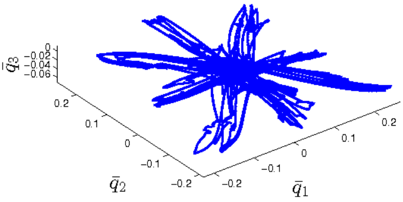
Simulations

- In this talk, potential control and optimal control simulations are compared with measured human head/eye movement data.
- For the potential control and optimal control simulations, the head data are projected on to the Donders' surface.
- The head moves between selected initial and final orientations of this projected trajectories.
- The eye starts from the straight direction and tracks a back calculated path such that the eye compensates for the slower head movement in order to keep the image on the retina stable.

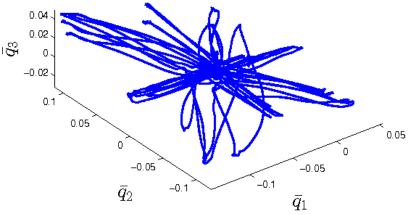
Simulations Contd

- Case I: The head and the eye track the measured human head/eye movement data.
- Caese II: The head optimally moves between the inital and final oritentations while the eye optimally tracks the back calculated path.
- Case III: The head moves between the initial and final orientations using a potential control while the eye tracks the back calculated path also using potential control.

Simulations Contd



(a) Eye trajectory in head coordinates as recorded experimentally by the laser dots.

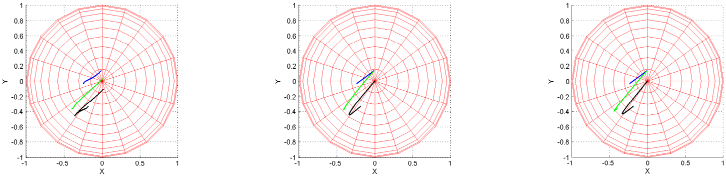


(b) Head trajectory in torso coordinates as recorded experimentally by the laser dots.

Figure: Eye and Head movement manoeuvres for a subject.

The continuous trajectory shown in the figure is broken into trajectory segments that show the desired behavior of eye/head complex moving in coordination to capture a target.

Simulation Contd



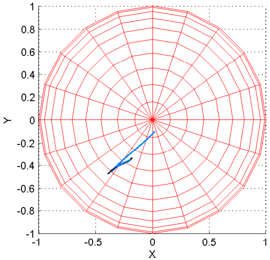
(a) Head and eye are tracking a known signal (Simulation I).

(b) Head moves optimally satisfying boundary constraints, while eye backtracking the head movement (Simulation II).

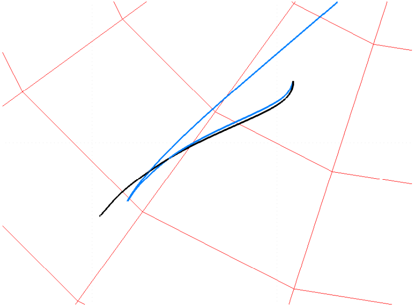
(c) Head moves potentially satisfying boundary constraints, while eye backtracking the head movement (Simulation III).

Figure: Trajectories of head and eye projected on the gaze (heading) space. Figs. 2b and 2c show simulated trajectories from dynamic models. Fig. 2a shows simulated trajectories when the eye and head closely tracks the experimentally measured data.

Eye Compensating for the Head: Actual Signal



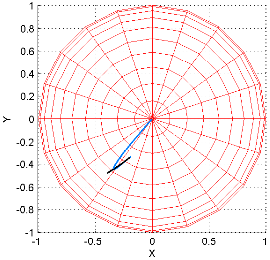
(a) Eye back tracking the computed trajectory (Tracking the actual trajectory)



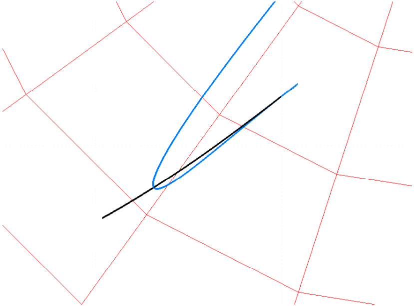
(b) closeup view

Figure: The head trajectories are not shown. The blue curves are the eye trajectories projected on the gaze space. The black curve is the computed trajectory $\xi(t)$ that the eye is expected to track, computed based on the head trajectory and the target position in order to keep the target stable on the retina.

Eye Compensating for the Head: Optimal Control



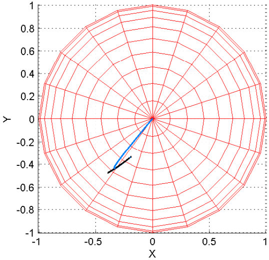
(a) Eye backtracking while the head moves optimally between initial and final point of a measured trajectory from a human subject.



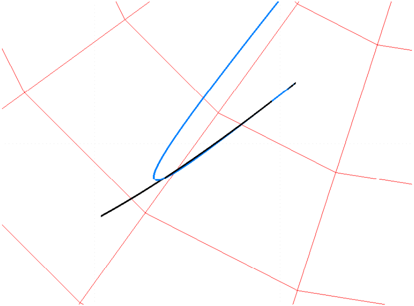
(b) closeup view

Figure: The head trajectories are not shown. The blue curves are the eye trajectories projected on the gaze space. The black curve is the computed trajectory $\xi(t)$ that the eye is expected to track, computed based on the head trajectory and the target position in order to keep the target stable on the retina.

Eye Compensating for the Head: Potential Control



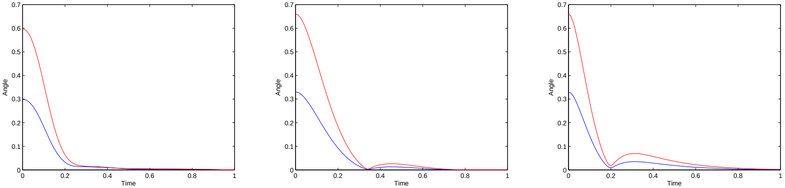
(a) Eye backtracking while the head moves potentially between initial and final point of a measured trajectory from a human subject.



(b) closeup view

Figure: The head trajectories are not shown. The blue curves are the eye trajectories projected on the gaze space. The black curve is the computed trajectory $\xi(t)$ that the eye is expected to track, computed based on the head trajectory and the target position in order to keep the target stable on the retina.

Eye Tracking Error



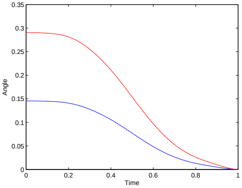
(a) Eye tracking error when head is following observed trajectory from a human subject.

(b) Eye tracking error when head is following optimal trajectory on the Donders' surface with constrained initial and final points.

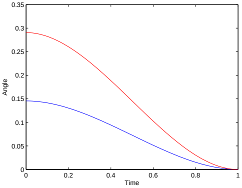
(c) Eye tracking error when head is potentially controlled towards a final point and is constrained by the Donders' surface.

Figure: Eye tracking error, in torso coordinates, has been expressed as angles. Blue: Angle between the eye quaternion and the target represented as a quaternion. Red: Angle between the eye direction and the target direction.

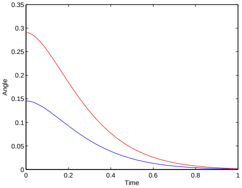
Head Tracking Error



(a) Head is following observed trajectory from a human subject.

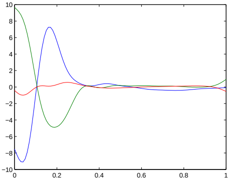


(b) Head is following optimal trajectory constrained by the Donders' surface with fixed initial and final conditions.

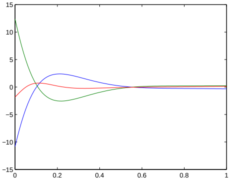


(c) Head follows a path of decreasing potential function towards a final point and is constrained by the Donders' surface.

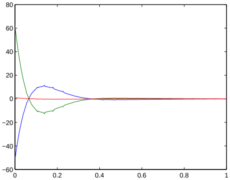
Figure: Head error between current and final position, in torso coordinates, has been expressed as angles. Blue: Angle between the 'Current Head Quaternion' and the 'Final Head Quaternion'. Red: Angle between the 'Current Head Direction' and the 'Final Head Direction'.



(a) Head and Eye are following a measured trajectory.



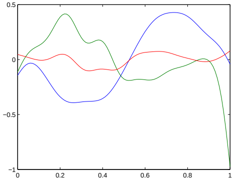
(b) Head is following an optimal trajectory where the end points are constrained and the Eye is backtracking the head.



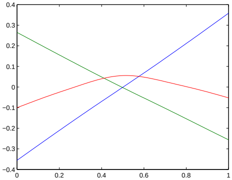
(c) Head is potentially controlled and the Eye is backtracking the head.

Figure: The figure displays three components of the torque applied to the eye dynamics. In Fig. 8a, the torque vector required for the eye dynamics to follow the observed trajectory is displayed. In Figs. 8b, 8c the optimal and the potential torques are respectively displayed.

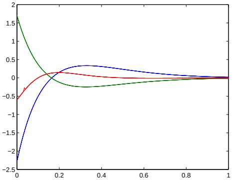
Head Torque Profile



(a) Head and Eye are following a measured trajectory.



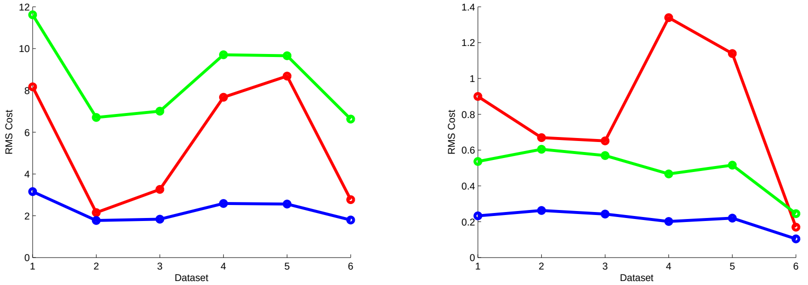
(b) Head is following an optimal trajectory where the end points are constrained and the Eye is backtracking the head..



(c) Head is potentially controlled and the Eye is backtracking the head.

Figure: The figure displays three components of the torque applied to the head dynamics. In Fig. 9a, the torque vector required for the head dynamics to follow the observed trajectory is displayed. In Figs. 9b, 9c the optimal and the potential torques are respectively displayed.

RMS Cost



(a) Cumulative RMS Cost of Eye Control.

(b) Cumulative RMS Cost of Head Control.

Figure: RMS Cost of head and eye control cumulated over multiple manoeuvres. Each of the six subjects are indexed from 1 to 6 in the x-axis. The 'blue' color is for the optimal control, 'red' color is for the actual control and the 'green' color is for the potential control.

Possible Improvements to the Model

- The head can be modeled as an arbitrarily shaped object that rotates about an eccentric pivot point.
- Muscle models can be introduced to the model.

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