1) For a binomial distribution, the standard deviation (σ) is given by,

$$\sigma = \sqrt{np(1-p)}$$

Variance (v) is the square of the standard deviation.

Therefore $v = np(1-p)$

Therefore if p is kept constant and n is increased, v increases.

Hence the answer is "true".

2) $P(0) = 0.35$

3) At least one retransmission implies $x \geq 1$. That is $x = 1, 2,$ or 3.

Therefore $P(x \geq 1) = P(1) + P(2) + P(3)$

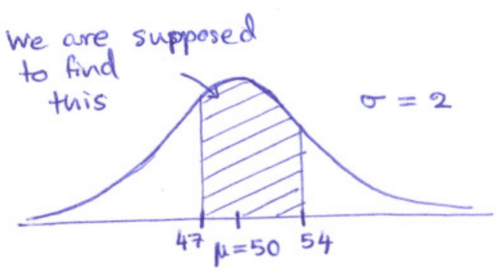
$= 0.35 + 0.25 + 0.05$

$= 0.65$

[Method II: $x \geq 1$ is the complement of $x = 0$.
Therefore $P(x \geq 1) = 1 - P(0) = 1 - 0.35 = 0.65$]

4) μ (mean) $= 0(0.35) + 1(0.35) + 2(0.25) + 3(0.05)$

$= 1$

5) 

$54 \Rightarrow z_1 = \frac{54-50}{2} = 2$

From the normal table, cumulative probability for $z_1 = 0.9772$

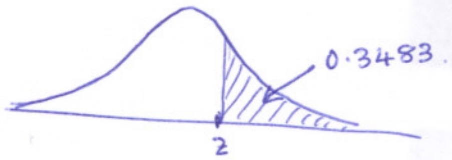
$47 \Rightarrow z_2 = \frac{47-50}{2} = -1.5$

From the table, the cumulative probability for $z_2 = 0.668$.

Therefore the probability of the shaded area $= 0.9772 - 0.0668$

$= 0.9104$

6) In this question, what we know is a probability and it is asking for a z-score. (It is asking for the z-score for which the area to its right is 0.3483)



The table gives the relation between the z-scores and the area to the left. Therefore, before looking up the table, we have to covert this to an area to the left.

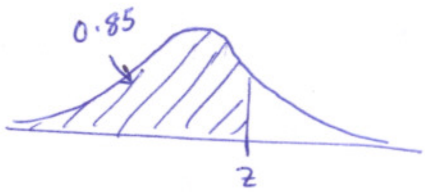
The area to the left of the z-score of interest $= 1 - 0.3483$
$= 0.6517$

Find the closest value for this in the middle of the table and read the corresponding z-score. (We have to search for 0.6517 in the middle since it is an area not a z-score)

We can find 0.6517 for z-score 0.39.

Therefore the answer is 0.39.

7). This question is similar to (6) in the sense that we know a probability and it is asking for a z-score.
(It is asking for the z-score for which area to the left of it is 0.85).



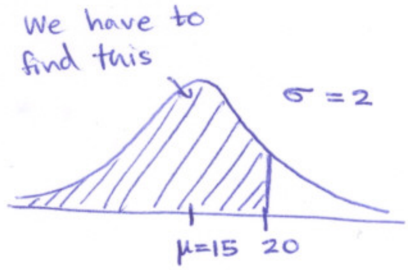
Because what we have is already an area to the left, we can directly search for it in the table.

(Again since 0.85 is an area, we have to search for the closest value to it in the middle of the table).

The closest value to 0.85 we can find in the table is 0.8508 which corresponds to 1.08.

Therefore the answer is 1.08.

8)



$20 \Rightarrow \quad z = \frac{20-15}{2} = 2.5$

The cumulative probability corresponding to $z = 2.5$ is 0.9938 (from the table).

Cumulative probability is the area to the left.

Therefore the probability of interest = 0.9938.

9) x – total number of tails.

0 HHHH	1 HHHT	1 HHTH	2 HHTT
1 HTHH	2 HTHT	2 HTTH	3 HTTT
1 THHH	2 THHT	2 THTH	3 THTT
2 TTHH	3 TTHT	3 TTTH	4 TTTT

Corresponding x value for each outcome is shown.

$P(X = 0) = \frac{1}{16} \qquad P(X = 1) = \frac{4}{16} = \frac{1}{4}$

Therefore the answer should be (D).

10) X – lesser value of the two

① (1,1)	① (1,2)	① (1,3)	① (1,4)	① (1,5)	① (1,6)
① (2,1)	② (2,2)	② (2,3)	② (2,4)	② (2,5)	② (2,6)
① (3,1)	② (3,2)	③ (3,3)	③ (3,4)	③ (3,5)	③ (3,6)
① (4,1)	② (4,2)	③ (4,3)	④ (4,4)	④ (4,5)	④ (4,6)
① (5,1)	② (5,2)	③ (5,3)	④ (5,4)	⑤ (5,5)	⑤ (5,6)
① (6,1)	② (6,2)	③ (6,3)	④ (6,4)	⑤ (6,5)	⑥ (6,6)

x values corresponding to each outcome is shown.

$P(X = 1) = \frac{11}{36} \qquad P(X = 2) = \frac{9}{36} = \frac{1}{4}$

Therefore, the answer is (A).

11) Y - product of the two numbers.

Values of Y should be single numbers. Therefore (A) cannot be an answer.

Y can have the value 1, (1×1), since (C) does not contain 1, it cannot be an answer.

Y cannot take the value 0 since the dice do not have 0. Therefore (D) cannot be an answer.

Therefore (B) is the answer.

12) X - number of siblings for a student.

According to the table this can have values from 0 thru 7.

Therefore the answer is (C).

13) $$\text{mean } (\mu) = 0 \cdot \left(\frac{7}{24}\right) + 1\left(\frac{13}{48}\right) + 2\left(\frac{3}{16}\right) + 3\left(\frac{7}{48}\right) + 4\left(\frac{1}{16}\right) + 5\left(\frac{1}{24}\right)$$

$$= 1.542$$

Therefore the answer is (B).

14) a.) Probability distribution of a random variable X is a table that lists all the possible values of X and the corresponding probabilities.

Therefore the answer for part a is

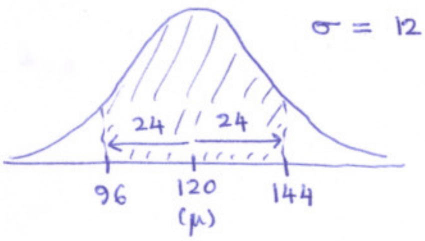
X	P(X)
0	0.09
1	0.22
2	0.26
3	0.20
4	0.12
5	0.11

b.) Expected value of a random variable is its mean.

Therefor the answer $= 0(0.09) + 1(0.22) + 2(0.26) + 3(0.20) + 4(0.12) + 5(0.11)$

$= 2.37$

15)



$24 = 2.\sigma$

Therefore the question is asking for the percentage of data that falls within two standard deviations of the mean.

Therefore from the emperical rule, this percentage is 95%.

Therefore the answer is (D)

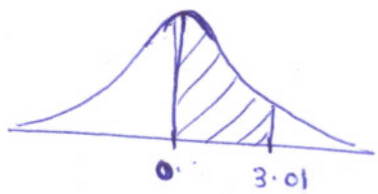
16) Using the emperical rule, the percentage of bulbs having a lifetime that lie within 1 standard deviation of the mean = 68%

The answer is (D)

17) (A) is not true since an area cannot be negative.
[z values to the left of z = 0 are negative. But not the corresponding areas].

Therefore the answer is (A).

18) Since we are talking about a <u>standard</u> normal curve, what is given are the z-scores.



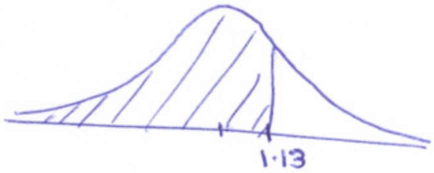
0 ⟶ area to the left = 0·5 (from the table)

3·01 ⟶ area to the left = 0·9987 (" " ")

Therefore the shaded area is 0·9987 − 0·5
= 0·4987

The answer is (B)

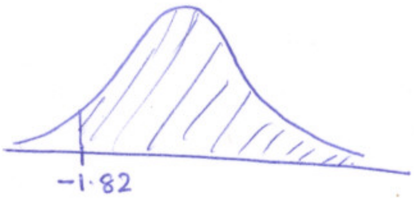
19) As in (18) what is given here is also a z-score



from the table, the area to the left of 1.13
= 0.8708

Therefore the answer is (C).

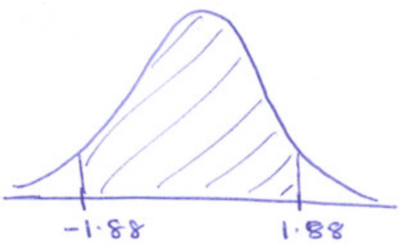
20)



From the table, the area to the left of −1.82
= 0.0344.

Therefore area to the right = 1 − 0.0344
= 0.9656

21)

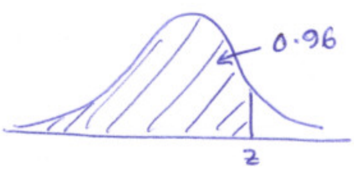


Area to the left of 1.88 = 0.9699
" " " " " −1.88 = 0.0301

Therefore the area inbetween = 0.9699 − 0.0301
= 0.9398

The answer is (C).

22) Like (6) and (7), we are given a area and asked to find a z-score.

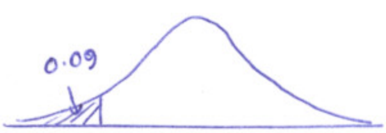


From the table z = 1.75.

(The closest we can find to 0.96 from the areas are 0.9599 which corresponds to z = 1.75).

The answer is (C)

23) This is similar to (22).

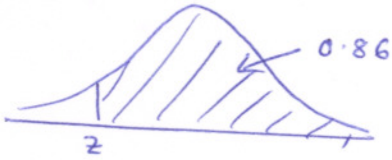


From the table z = −1.34.

(the closest area to 0.09 is 0.0901 which corresponds to −1.34).

The answer is (E).

24) This is similar to (6), (7), (22) and (23) in the sense that we are given a area and asked to find a z-score



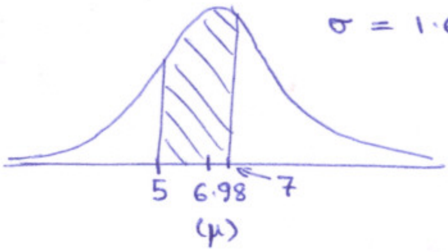
~~The closest to 0.86 we can find in the areas~~
Before looking at the table we have to convert this to an area to the left.

Area to the left of z-score of interest $= 1 - 0.86 = 0.14$.

The closest we can find, to this area from the table is 0.1401 which corresponds to the z-score -1.08.

Therefore the answer is -1.08.

25) Similar to (5).



$\sigma = 1.62$

(Remember to read <u>0.01</u> from the table. Not <u>0.10</u>.)

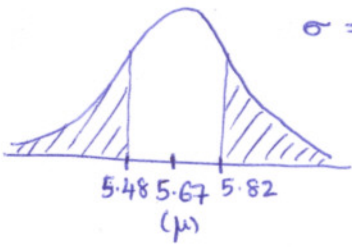
$7 \Rightarrow z_1 = \dfrac{7 - 6.98}{1.62} = 0.0123 \approx 0.01 \xrightarrow{\text{table}}$ ~~area to~~ Cumulative probability $= 0.5040$

$5 \Rightarrow z_2 = \dfrac{5 - 6.98}{1.62} = -1.22 \xrightarrow{\text{table}}$ Cumulative probability $= 0.1112$.

Therefore the probability of interest $= 0.5040 - 0.1112$
$= 0.3928$

Therefore the answer is (C).

26)

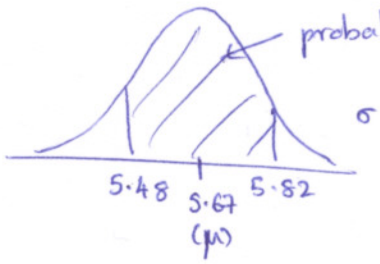


$\sigma = 0.070$ g

Since the problem is asking for the probability of a quater getting rejected, it is the region that the quater is not accepted. This area is shaded in the figure.

One way to find this probability is to find the probability of a quater getting accepted (un shaded area in the figure) and do (1 – what you get).

The problem of find the probability that the quater gets accepted is similar to (25), (5).



$5.82 \Rightarrow z_1 = \dfrac{5.82 - 5.67}{0.07} = 2.14 \longrightarrow$ cumulative probability $= 0.9838$

$5.48 \Rightarrow z_2 = \dfrac{5.48 - 5.67}{0.07} = -2.71 \longrightarrow$ " " $= 0.0034$

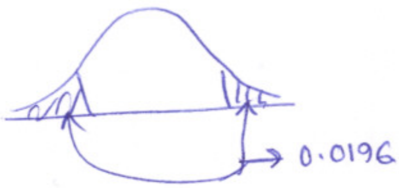
Therefore the probability that a quater getting accepted $= 0.9838 - 0.0034$

$= 0.9804$

Therefore the probability that a quater <u>not</u> getting accepted

$= 1 - 0.9804$

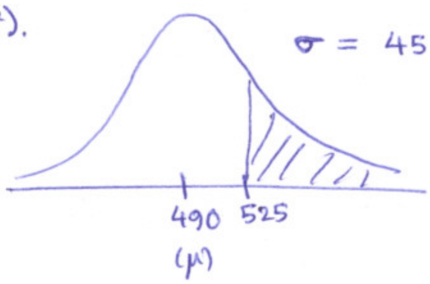
$= 0.0196$



This expressed as a percentage $= 1.96\%$

Therefore the answer is (C).

27).



$525 \Rightarrow z = \dfrac{525 - 490}{45} = 0.777\ldots \approx 0.78.$

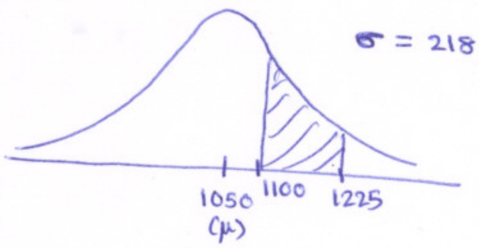
From the table the probability to the left $= 0.7823$

Therefore the probability to the right $= 1 - 0.7823$

$= 0.2177$

The answer is (B).

28) Similar to (25) and (5).



1225 $\Rightarrow$ $z_1 = \dfrac{1225 - 1050}{218} = 0.80$

$\Rightarrow$ area to the left $= 0.7881$

1100 $\Rightarrow$ $z_2 = \dfrac{1100 - 1050}{218} = 0.23$

$\Rightarrow$ area to the left $= 0.5910$

Therefore the probability of interest $= 0.7881 - 0.5910$

$= 0.1971$

The answer is (D).

29) Since we are trying to count cards drawn in two suits, the problem cannot be defined to have only two possible outcomes.

Therefore the answer is (A)

30) Since this is a binomial distribution, we have to use the equation for the mean of a binomial distribution. That is

$\mu = n.p.$ n - number of trials. p - probability of success.

$n = 16$.

X - counts the number of students working full time.

Therefore my "success" is "working full time".

therefore the probability of success $p = 0.22$.

Therefore $\mu = 16(0.22) = 3.52$

The answer is (D).

31) X - counts the number of correct answers.

Therefore if "success" is defined to be "getting it correct", the other possibility is "not getting correct". Therefore for each trial, there are only two outcomes.

~~Therefore~~
Getting one question correct or not is independent of getting others correct or not. Therefore the trials are independent.

Therefore, this can be modelled by a binomial distribution.

Then $n = 16$.

p - is the probability of success. that is the probability of getting a question correct.

Since one question has four answers and only one of them is right, the probability of getting it correct $= \frac{1}{4}$

Therefore $p = \frac{1}{4}$.

The mean (μ) and the standard deviation (σ) of a binomial distribution are given by

$$\mu = n \cdot p \qquad \sigma = \sqrt{n \cdot p(1-p)}.$$

Therefore $\mu = 16 \cdot \frac{1}{4} = \boxed{4}$ ← mean.

$$\sigma = \sqrt{16 \cdot \frac{1}{4} \cdot \left(1 - \frac{1}{4}\right)} = \sqrt{16 \cdot \frac{1}{4} \cdot \frac{3}{4}} = \sqrt{3} = \boxed{1.732}$$ ← standard deviation.

33) We are trying to find the probability of getting exactly 3 first serves

Therefore if X - number of successful first serves,

"success" is getting a first serve

"failure" is not gettin a first serve.

(Only two outcomes.)

Each serve is independent of the others.

Therefore X - is a binomial random variable.

We are supposed to find $P(X=3)$.

The probability of a binomial random variable is given by

$$P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

So if we know n, p and x we can find the probability.

n = total number of trials. $n = 7$.

p = probability of success
= probability of getting a first serve $p = 0.59$

x = number of successful first serves we are interested in $x = 3$

Therefore $P(X=3) = \frac{7!}{3!\,4!} 0.59^3 (1-0.59)^{7-3}$

$= 35 \cdot 0.59^3 \cdot 0.41^4$

$= 0.2031$

The answer is (E)

34) Same as (33) with "success" = someone being left handed.

$n = 6$

$p = 0.11$

$x = 2$.

$$P(x=2) = \frac{6!}{2!\,4!} (0.11)^2 (1-0.11)^{6-2}$$

$$= 15 \cdot (0.11)^2 (0.89)^4$$

$$= 0.1139$$

The answer is (A).

35) Same as (33) and (34) with "success" = making a shot.

$n = 3$

$p = 0.7$

$x = 3$ because we are interested in getting all 3 shots.

therefore

$$P(x=3) = \frac{3!}{3!\,0!} 0.7^3 (1-0.7)^{3-3}$$

$$= 1 \cdot 0.7^3 (0.3)^0$$

$$= 0.343$$

the answer is (B).