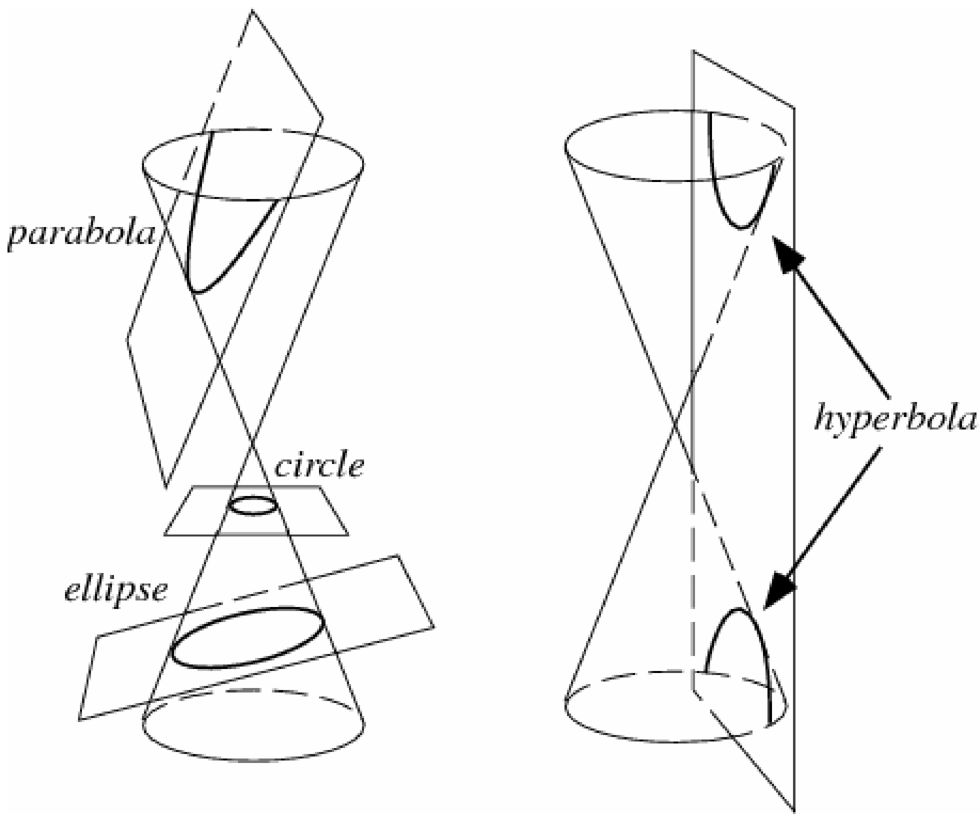
Conic Sections



Property Summary of Standard ("center" at origin) Conic Sections

	Circle	Ellipse	Parabola		Hyperbola	
			Vertical	Horizontal	Horizontal	Vertical
Standard Equation	$x^2+y^2=r^2$	$\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$	$x^2=4py$	$y^2=4px$	$\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$	$\dfrac{y^2}{b^2}-\dfrac{x^2}{a^2}=1$
Center	$(0,0)$	$(0,0)$	–	–	$(0,0)$	$(0,0)$
Foci	$(0,0)$	$\begin{cases}(\pm\sqrt{a^2-b^2},0) & \text{if } a>b\\ (0,\pm\sqrt{b^2-a^2}) & \text{if } b>a\end{cases}$	$(0,p)$	$(p,0)$	$(\pm\sqrt{a^2+b^2},0)$	$(0,\pm\sqrt{a^2+b^2})$
Directrix	–	–	$y=-p$	$x=-p$	–	–
Vertices	$(0,\pm r)$, $(\pm r,0)$	$(0,\pm b)$, $(\pm a,0)$	$(0,0)$	$(0,0)$	$(\pm a,0)$	$(0,\pm b)$
Major Axis Length	$2r$	$\begin{cases}2a & \text{if } a>b\\ 2b & \text{if } b>a\end{cases}$	–	–	–	–
Minor Axis Length	$2r$	$\begin{cases}2b & \text{if } a>b\\ 2a & \text{if } b>a\end{cases}$	–	–	–	–
Transverse Axis Length	–	–	–	–	$2a$	$2b$
Conjugate Axis Length	–	–	–	–	$2b$	$2a$
Asymptotes	–	–	–	–	$y=\pm\dfrac{bx}{a}$	$y=\pm\dfrac{bx}{a}$
Eccentricity	0	$\begin{cases}\dfrac{\sqrt{a^2-b^2}}{a} & \text{if } a>b\\ \dfrac{\sqrt{b^2-a^2}}{a} & \text{if } b>a\end{cases}$	1	1	$\dfrac{\sqrt{a^2+b^2}}{a}$	$\dfrac{\sqrt{a^2+b^2}}{b}$

Shifted Conic Sections

	Circle	Ellipse	Parabola		Hyperbola	
			Vertical	Horizontal	Horizontal	Vertical
Standard Equation	$(x-h)^2+(y-k)^2=r^2$	$\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$	$(x-h)^2=4p(y-k)$	$(y-k)^2=4p(x-h)$	$\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$	$\dfrac{(y-k)^2}{b^2}-\dfrac{(x-h)^2}{a^2}=1$
Center	(h,k)	(h,k)	–	–	(h,k)	(h,k)
Foci	(h,k)	$\begin{cases}(h\pm\sqrt{a^2-b^2},k) & \text{if } a>b\\(h,k\pm\sqrt{b^2-a^2}) & \text{if } b>a\end{cases}$	$(h,k+p)$	$(h+p,k)$	$(h\pm\sqrt{a^2+b^2},k)$	$(h,k\pm\sqrt{a^2+b^2})$
Directrix	–	–	$y=k-p$	$x=h-p$	–	–
Vertices	$(h,k\pm r)$, $(h\pm r,k)$	$(h,k\pm b)$, $(h\pm a,k)$	(h,k)	(h,k)	$(h\pm a,k)$	$(h,k\pm b)$
Major Axis Length	$2r$	$\begin{cases}2a & \text{if } a>b\\2b & \text{if } b>a\end{cases}$	–	–	–	–
Minor Axis Length	$2r$	$\begin{cases}2b & \text{if } a>b\\2a & \text{if } b>a\end{cases}$	–	–	–	–
Transverse Axis Length	–	–	–	–	$2a$	$2b$
Conjugate Axis Length	–	–	–	–	$2b$	$2a$
Asymptotes	–	–	–	–	$y=k\pm\dfrac{b(x-h)}{a}$	$y=k\pm\dfrac{b(x-h)}{a}$
Eccentricity	0	$\begin{cases}\dfrac{\sqrt{a^2-b^2}}{a} & \text{if } a>b\\\dfrac{\sqrt{b^2-a^2}}{a} & \text{if } b>a\end{cases}$	1	1	$\dfrac{\sqrt{a^2+b^2}}{a}$	$\dfrac{\sqrt{a^2+b^2}}{b}$

What you need to do... Given a quadratic function of x and y,

- Identifying the conic:

  a. If it does EITHER y^2 term OR x^2 term, NOT BOTH, then it is a **parabola**.

  b. If the y^2 term and the x^2 term,both have the SAME COEFFICIENT, then it MAY BE a **circle**.

  c. If the y^2 term and the x^2 term,both have the DIFFERENT COEFFICIENTS, but the SAME SIGN, then it MAY BE an **ellipse**.

  d. If the y^2 term and the x^2 term has OPPOSITE SIGNS, then it is an **hyperbola**.

- Finding key values of the conic:

  Use "completing the square" or any other necessary operations to convert to a standard form listed above.

Examples

1. Identify the conic given by $4x^2 + 9y^2 = 36$

   Since the coefficients of the x^2 and y^2 terms have coefficients of unequal magnitude but same sign, this is an ellipse.

   Divide both sides by 36 to obtain $\left(\frac{4}{36}\right)x^2 + \left(\frac{9}{36}\right)y^2 = 1$.

   Then simplify a bit, $\left(\frac{1}{9}\right)x^2 + \left(\frac{1}{4}\right)y^2 = 1$.

   This is simply, $\dfrac{x^2}{3^2} + \dfrac{y^2}{2^2} = 1$.

   Note that $a > b$ and $\sqrt{a^2 - b^2} = \sqrt{3^2 - 2^2} = \sqrt{9 - 4} = \sqrt{5}$

   So, the key values of this ellipse are:

   - Center: $(0, 0)$
   - Foci: $(\pm\sqrt{5}, 0)$
   - Ends of major axes: $(-3, 0)$ and $(3, 0)$
   - Ends of minor axes: $(0, -2)$ and $(0, 2)$

2. Identify the conic given by $4x^2 + 8x + 4y^2 + 24y - 9 = 0$

   Since the coefficients of x^2 and y^2 are the same, this may be a circle.

   Group the x and y terms and factor out the coefficients of x^2 and y^2, and also add 9 to both sides: $4(x^2+2x)+4(y^2+6y) = 9$

   Complete the square: $4\left(x^2 + 2x + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2\right) + 4\left(y^2 + 6y + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2\right) = 9$

   Simplify: $4(x^2 + 2x + 1 - 1) + 4(y^2 + 6y + 9 - 9) = 9$

   Simplify more: $4(x^2 + 2x + 1) - 4 + 4(y^2 + 6y + 9) - 36 = 9$

   Simplify more: $4(x^2 + 2x + 1) + 4(y^2 + 6y + 9) - 40 = 9$

   Group the square terms we made: $4(x + 1)^2 + 4(y + 3)^2 - 40 = 9$

   Add 40 to both sides: $4(x + 1)^2 + 4(y + 3)^2 = 49$

   Divide both sides by 4 to finally get: $(x + 1)^2 + (y + 3)^2 = \frac{49}{4}$

   Which is: $(x + 1)^2 + (y + 3)^2 = \left(\frac{7}{2}\right)^2$

   Since the right hand side is a positive number, this IS a circle.

   So, the key values of this circle are:

   - Center: $(-1, -3)$
   - Radius: $\frac{7}{2}$