

1. Laws of real numbers

Let a and b be two real numbers then,

- (a) $0 \cdot a = 0$
- (b) $-a = (-1) \cdot a$
- (c) $-(-a) = a$
- (d) $a \cdot (-b) = -(ab)$
- (e) $(-a) \cdot (-b) = ab$

2. Laws of exponents

Let a and b be two real numbers, and n and m be two integers or rational numbers

- (a) $a^m a^n = a^{m+n}$
- (b) $(a^m)^n = a^{mn}$
- (c) $(ab)^m = a^m b^m$
- (d) if $a \neq 0$, then $\frac{a^m}{a^n} = a^{m-n}$
- (e) if $b \neq 0$, then $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
- (f) if $a \neq 0$, then $a^0 = 1$

3. Laws of principal roots

Let a and b be two real numbers, and n and m be two integers,

- (a) $\sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$
- (b) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
- (c) $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a}$
- (d) i. If n is even $\sqrt[n]{x^n} = |x|$
ii. If n is odd $\sqrt[n]{x^n} = x$

4. Absolute value

(a) *Geometric interpretation:* The absolute value of a real number x , denoted by $|x|$, is the distance from the origin to x (or x to the origin), regardless of the direction.

(b) *Algebraic interpretation:*

$$|x| = \begin{cases} x & \text{when } x \geq 0 \\ -x & \text{when } x < 0 \end{cases}$$

5. Properties of absolute value

- (a) For any real number x , we have
 - i. $|x| \geq 0$
 - ii. $x \leq |x|$ and $-x \leq |x|$
 - iii. $|x^2| = x^2$
- (b) For two real numbers a and b ,
 - i. $|ab| = |a||b|$ and if $b \neq 0$,
 $\frac{|a|}{|b|} = \left|\frac{a}{b}\right|$

ii. $|a + b| \leq |a| + |b|$ (Triangle inequality)

6. The distance between two numbers a and b on the real line is given by $|a - b|$ and this is equal to $|b - a|$.

7. Distance between two points in rectangular coordinates

Given two points $P(x_P, y_P)$ and $Q(x_Q, y_Q)$, distance d is:

$$d = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2}$$

8. **The mid point rule** The mid point M of the line segment PQ

$$M = \left(\frac{x_P + x_Q}{2}, \frac{y_P + y_Q}{2} \right)$$

9. **Point dividing a line to a given ratio** The point R on the line segment PQ such that $PR/QR = p/q$...

10. Slope of a line

Given two points P and Q , slope m of line through P and Q :

$$m = \frac{y_Q - y_P}{x_Q - x_P}$$

11. x - and y - intercepts ...

12. Equation of lines

- (a) *Slope - y - intercept formula:* Given slope m and y -intercept c : $y = mx + c$
- (b) *Slope - point formula:* Given slope m and point $P(x_P, y_P)$:
 $y = m(x - x_P) + y_P$
- (c) *Two point formula:* Given two points $P(x_P, y_P)$ and $Q(x_Q, y_Q)$: ...
- (d) *Vertical and horizontal lines:*
- (e) *Standard Equation* $Ax + By + C = 0$

13. **Parallel and perpendicular lines** Given two lines with slopes m_1 and m_2 ,

- (a) They are parallel if $m_1 = m_2$
- (b) They are perpendicular if $m_1 = -1/m_2$

14. **Symmetry** Given the equation of a graph:

- (a) *Symmetric about x -axis* if replacing y with $-y$ yields an equivalent equation.
- (b) *Symmetric about y -axis* if replacing x with $-x$ yields an equivalent equation.
- (c) *Symmetric about the origin* if replacing x with $-x$ and y with $-y$ yields an equivalent equation.

15. **Standard formula of a circle** Given center (h, k) and radius r ,

$$(x - h)^2 + (y - k)^2 = r^2$$

16. **Zero-product property of real numbers:** For two real numbers p and q , we get $pq = 0$ if and only if $p = 0$ or $q = 0$ or both

17. Quadratic Formula

The solution to a quadratic equation $ax^2 + bx + c = 0$, when $a \neq 0$ is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

18. Scientific notation ...

19. **Significant digits, decimal places and rounding ... (A-1)**

20. Factoring ...

21. **Open and closed intervals, mixed (half open/closed) intervals, unbounded intervals ...**

22. **Completing the square ...** "*adding $(B/2)^2$...*"

23. Using the discriminant ...

24. **Solving different types of equations and inequalities...** (*chapter 2*)

25. **Rewriting expressions removing the absolute value sign ...**