MATH1550: Precalculus

Lecture 01

August 26, 2010

Best wishes for a great semester ahead!

Numbers in day to day life

- Counting Numbers (a.k.a. Natural Numbers)
  - Numbers that use to count things
    e.g. counting people, votes, horses, stars etc.
    1, 2, 50, 102356486, etc.
    0 is a tricky customer .... some use it and some don't,
    the book does NOT
- Integers
  - Signed natural numbers along with 0
    e.g. monetary transactions, like using gold coins
    $(+)$ for assets; $(-)$ for liabilities or debt
    0, 1, -22, -150, 78545234130, -698524 etc.
- Rational Numbers
  - Numbers that can be expresses as a fraction "ratio" of two integers (hence the name), with non zero denominator.
    e.g. dividing 3 similar cakes equally among 15 people,
    $\frac{1}{2}$, $0.75(=\frac{3}{4})$, $4.1\overline{62}$, $15(=\frac{15}{1})$, etc.
    Integers are rational numbers themselves
    Decimal expansion will either terminate or repeats after a while

Other types of numbers

There are other types of numbers such as:

- Irrational numbers
  Numbers which CANNOT be expressed as a ratio of two integers (hence the name)
- Real numbers
  Rational numbers and irrational numbers together
- Complex numbers
  ... a little "complex" if you have not see them before... :)

Constants and Variables

"A **constant** is a value that we know for sure or does not change its value within our range of consideration"

"A **variable**, on the other hand is some thing we do not know the value for sure within our range of consideration"

"The legitimate range of values a variable can assume is known as the **domain** of the variable"

Examples for constants and variables

Since we do not know the exact value of a variable, we think of it as an “unknown” number, and usually give it a name or use some symbol to represent the variable.

For example, we call a variable “Sam”, $\square$, $\circ$, x, α, y_1 etc. But we usually prefer to use roman letters like x, y, z.

If we write a mathematical expression, for example

$$\square + 5;$$

$\square$ is the variable and 5 is the constant.
If we put 7 in the place of the box we can evaluate the expression to be $7 + 5 = 12$,
if we plug in -1236548 to box we get $-1236548 + 5 = -1236543$.

We would often write this as $x + 5$.

Linear Polynomials

Consider the mathematical expression $ax + b$. You are told that a and b are constant and x is variable. These types of expressions are called linear polynomials.

By fixing the values of a and b we get different linear polynomials.

For example, if we set $a = 3$ and $b = 7$, we get $3x + 7$, which is one linear polynomial,
If we set $a = 1$ and $b = -2$, we get $1x + (-2)$, which is one linear polynomial, this one we prefer to write simply as $x - 2$.

Quadratic Polynomials

The mathematical expression of the form $ax^2 + bx + c$ and again you are told that a b and c are constant and x is variable; note that both the variable and its square are in the expression. These types of expressions are called **quadratic** polynomials.

By fixing the values of a b and c we get different quadratic polynomials.

For example, by setting $a = 5$ $b = -2$ and $c = 1$ we get $5x^2 - 2x + 1$ ...

Try it your self....

Identify the type, constants and variables in the following expressions

1. $5x + 2$
2. $x^2 + 6x - 5$
3. $2t^2 + 3t + 6$
4. $qu + pu^2 + r$ (you are told that u is the variable)
5. $y^2 + 6$

Answers

Recall that $ax + b$ is a Linear polynomial and $ax^2 + bx + c$ is a Quadratic polynomial

1. $5x + 2$
   Linear, variable x, constants $a = 5,\ b = 2$
2. $x^2 + 6x - 5$
   Quadratic, variable x, constants $a = 1,\ b = 6,\ c = -5$
3. $2t^2 + 3t + 6$
   Quadratic, variable t, constants $a = 2,\ b = 3,\ c = 6$
4. $qu + pu^2 + r$
   Quadratic, variable u, constants $a = p,\ b = q,\ c = r$
5. $y^2 + 6$
   Quadratic, variable y, constants $a = 1,\ b = 0,\ c = 6$

General Polynomials

We can consider expressions which contains higher powers of a given variable, say 'x' in the form $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$, where, a_n, a_{n-1}, $\ldots$ a_1, a_0 are all constants.

The highest power of the variable is called the **degree** of the polynomial.

For example:
A linear polynomial is a polynomial of degree 1
A quadratic polynomial is a polynomial of degree 2
$x^3 + 2x^2 + 3$ is a polynomial of degree 3

Try it your self....

Identify the degree and constants of the following polynomials

1. $5x^4 + 6x^3 + 3x^2 + 2x + 2$
2. $-3x^6 + 9x^3 + 3x - 1$
3. $2t^2 + 3t$
4. $3x + 4x^3 + 2x^2 + 5x^4 + 1$
5. $3x^2 + 4x + 2x + 5 + 2$

Answers

Compare with $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

1. $5x^4 + 6x^3 + 3x^2 + 2x + 2$
   Degree $= 4$, $a_4 = 5$, $a_3 = 6$, $a_2 = 3$, $a_1 = 2$, $a_0 = 1$
2. $-3x^6 + 9x^3 + 3x - 1$
   Degree $= 6$, $a_6 = -3$, $a_5, a_4 = 0$, $a_3 = 9$, $a_2 = 0$, $a_1 = 3$, $a_0 = -1$
3. $2t^2$
   Degree $= 2$, $a_2 = 2$, $a_1 = 0$, $a_0 = 0$
4. $3x + 4x^3 + 2x^2 + 5x^4 + 1$
   Degree $= 4$, $a_4 = 5$, $a_3 = 4$, $a_2 = 2$, $a_1 = 3$, $a_0 = 1$
5. $3x^2 + 4x + 2x + 5 + 2$
   First simplify the polynomial $3x^2 + 6x + 7$
   Degree $= 2$, $a_2 = 3$, $a_1 = 6$, $a_0 = 7$

Evaluating Polynomials

Just like in our "$\Box$" example earlier, given a polynomial $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$, and the value of the variable x, you have to put the given value in every place x appears.

Evaluate the following polynomials at the given values

1. $2x^2 + 3x + 1$ at 3 , t, $\diamondsuit$
2. $2x^3 - 3x + 1$ at 1, $\spadesuit$, -2
3. $4x - 3$ at $3/4$, $x + h$, 0
4. $3x^2 + x - 4$ at 4, $x + h$, x^2

QUIZ

QUIZ TIME...

Instructions: Please DO NOT write your name on the answer paper.

This is just a survey.

1. What is your current standing in the university? (Freshman, Sophomore, Junior or Senior)
2. Why are you taking this class? (Required/Recommended/Good for the career/As a prerequisite for calculus/FOR FUN, etc)
3. What are your comments about today's class?