MATH1550: Precalculus

Lecture 02

August 27, 2010

Recap of last class

We talked about

1. Natural numbers, integers, rational numbers, irrational numbers and real numbers
2. *Variables* and *constants*
3. Introduced linear, quadratic and general polynomials $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

My plan for the next few days

I hope review some important concepts, mostly the material in Appendices of the book. But I may not follow the same order, in particular, I will go through Appendix B, then come back to A. I may add a few more review topics on top of what is given in the book.

Missed out in last class ...
Substituting "values" to a variable

We can give a name to a mathematical expression of a variable, similar to naming variables. For example we may write

$p(x) = 2x^2 + 4x + 6;$

- p is the actual name of the expression
- the parenthesized x denotes the variable.

If we write $p(2)$, it means, we want to **evaluate** $p(x)$ at $x = 2$. This means, we need to "plug in" (called **substituting**) 2 in place all x's in the expression.

For our example, therefore,
$p(2) = 2(2)^2 + 4(2) + 6 = (2 \times 4) + (4 \times 2) + 6 = 8 + 8 + 6 = 22$

Now try it yourself...

Given $p(x) = 2x^2 + 4x + 6$ find the following

1. $p(\spadesuit)$
2. $p(\diamondsuit + \spadesuit)$
3. $p(y + z)$
4. $p(y + 1)$
5. $p(x^2)$
6. $p(x + h)$

Answers

Given $p(x) = 2x^2 + 4x + 6$:

1. $p(\spadesuit) = 2\spadesuit^2 + 4\spadesuit + 6$
2. $p(\diamondsuit + \spadesuit) = 2(\diamondsuit + \spadesuit)^2 + 4(\diamondsuit + \spadesuit) + 6$
3. $p(y + z) = 2(y + z)^2 + 4(y + z) + 6$;
   We can simplify this a bit more...
   $= 2(y + z)(y + z) + 4(y + z) + 6$
   $= 2(y \cdot y + y \cdot z + z \cdot y + z \cdot z) + 4(y + z) + 6$
   $= 2(y^2 + 2yz + z^2) + 4y + 4z + 6$
   $= 2y^2 + 4yz + 2z^2 + 4y + 4z + 6$
4. $p(y + 1) = 2(y + 1)^2 + 4(y + 1) + 6$ Let's be clever here!
   This is similar to the previous one... but with z replaced by 1.
   We can get the answer quickly (from the previous one), by substituting 1 to z... :)
   $= 2y^2 + 4y(1) + 2(1)^2 + 4y + 4(1) + 6$
   $= 2y^2 + 4y + 2 + 4 + 6 = 2y^2 + 4y + 12$

Answers continued...

Given $f(x) = 2x^2 + 4x + 6$:

5. $p(x^2) = 2(x^2)^2 + 4(x^2) + 6 = 2x^4 + 4x^2 + 6$

6. $p(x+h) = 2(x+h)^2 + 4(x+h) + 6$
   $= 2(x+h)(x+h) + 4(x+h) + 6$
   $= 2(x \cdot x + x \cdot h + h \cdot x + h \cdot h) + 4(x+h) + 6$
   $= 2(x^2 + 2hx + h^2) + 4x + 4h + 6$
   $= 2x^2 + 4hx + 2h^2 + 4x + 4h + 6$

A failsafe method for Substituting values to a variable

Given $p(x) = 2x^2 + 4x + 6$; find $p(y + z)$

Replace x by the whole term you have to substitute, **including the parenthesis/brackets**.

This will avoid a lot of confusion!

A possible mistake:
$p(y + z) = 2y + z^2 + 4y + z + 6$ WRONG!

With the trick above:
$p(y + z) = 2(y + z)^2 + 4(y + z) + 6$ CORRECT!

Remember the following rules...

Let a and b be two real numbers then,

1. $0 \cdot a = 0$
2. $-a = (-1) \cdot a$
3. $-(-a) = a$
4. $a \cdot (-b) = -(ab)$
5. $(-a) \cdot (-b) = ab$

Review of Exponents

Given a number a and exponent n, i.e. a^n, it means ...

- If n is a positive integer $(1, 2, 3, \dots)$,

$$a^n = \underbrace{a \cdot a \cdot a \dots a \cdot a}_{n\text{-times}}$$

- If $n = -1$

$$a^{-1} = \frac{1}{a}$$

- If n is a positive integer $(1, 2, 3, \dots)$,

$$a^{-n} = \frac{1}{a^n} = \frac{1}{\underbrace{a \cdot a \cdot a \dots a \cdot a}_{n\text{-times}}}$$

(so that $-n$ is a negative integer)

Zero Exponent

For any number a, we define $a^0 = 1$, as long as $a \neq 0$

0^0 is undefined!

Remember the following rules...

Let a and b be two real numbers, and n and m be two integers,

1. $a^m a^n = a^{m+n}$
2. $(a^m)^n = a^{mn}$
3. $\dfrac{a^m}{a^n} = a^{m-n}$
4. $(ab)^m = a^m b^m$

A Practical Use of Exponents: Scientific Notation

The scientific notation is a very convenient way to represent very large or very small numbers.

For example, $0.00000001235 = 1.235 \times 10^{-8}$

or, $1235000000000 = 1.235 \times 10^{12}$

Review of n^{th} Roots

Given a real number a, the "n^{th} root of a" is a number b such that

$$b^n = a$$

Then we call b to be an n^{th} root of a.

- Both $1/4$ and $-1/4$ are *square* roots (i.e. the second roots) of $1/16$, because, $4^2 = 16$ and $(-4)^2 = 16$.
- 3 is a *cube* roots (i.e. the third root) of 27, because, $3^3 = 27$; in fact, 3 is the only real cube root of 27.
- -0.2 is a 5^{th} root -0.00032, because, $(-0.2)^5 = -0.00032$; in fact, -0.2 is the only real 5^{th} root of -0.00032.
- Both 2 and -2 are 4^{th} roots (i.e. the second roots) of 16, because, $2^4 = 16$ and $(-2)^4 = 16$. In fact, these are the only two real 4^{th} roots of 16
- -4 does not have a real square root because there is no real number a such that $a^2 = -4$

Principal n^{th} Root

We can make a few observations from the previous examples:

- Negative real numbers does not have real even roots
- Even numbered roots of positive real numbers always occur in pairs; one positive and one negative
- (Real valued) odd numbered roots are unique, and odd numbered root of a positive number is positive, and and odd numbered root of a negative number is negative.

To avoid the ambiguity of the even numbered roots of positive real numbers, we introduce the notion of **principal** n^{th} root. The principal n^{th} root, for even n, is simply the positive n^{th} root. For other cases, there is no ambiguity. So, the principal root and the real root are the same.

The principal n^{th} root is denoted by the "radical" sign $\sqrt[n]{\quad}$
The principal square root is denoted just by $\sqrt{\quad}$

Principal n^{th} Root: Examples

- $\sqrt{25} = 5$ because $5^2 = 5 \times 5 = 25$, and 5 is the principal square root of 25.
- Eventhough $(-5)^2 = 25$, it is WRONG to write $\sqrt{25} = -5$ according to the definition of $\sqrt{}$; since -5 is not the principal square root of 25.
- $\sqrt[4]{16} = 2$ because $2^4 = 16$, and since 2 is the principal 4^{th} root of 16.
- Eventhough $(-2)^4 = 16$, it is WRONG to write $\sqrt[4]{16} = -2$ according to the definition of $\sqrt[4]{}$; since -2 is not the principal 4^{th} root of 16.
- $\sqrt[3]{-125} = -5$ because $(-5)^4 = 125$, and it is unique.