MATH1550: Precalculus

Lecture 05

Tuesday, September 01, 2010

Recap of last class

We discussed about the *number line* and *absolute values*.

Plan for today

1. Complete the discussion on absolute values with some examples.
2. Solving equations
    1. Review of linear equations
    2. Equations reducible to linear equations
    3. Review of quadratic equations

Solving Equations

Given an equation, $p(x) = q(x)$, where $p(x)$ and $q(x)$ are two mathematical expressions of the variable x, we have to do some mathematical operations on the two sides of the equation, so that we maintain the **balance** (equality) and ultimately end up with a solution in the form $x = a$, where a does not contain the variable x.

Trivial example: Given $2x - 6 = 0$, we can first add 6 to both sides to get $2x = 6$ and then divide both sides by 2 to end up with $x = 3$. This is the **solution** of the equation.

Procedures that maintain the balance of equations

1. Adding or subtracting the **same quantity** on both sides
2. Multiplying or dividing both sides by the **same NON ZERO factor**
3. Simplifying an expression on either side

Some example to try - Verifying Solutions

1. $4x - 3 = 2x - 1 \quad ; x = 1,\ x = 0$
2. $\dfrac{1}{x} + \dfrac{1}{4-x} = 1 \quad ; x = 1,\ x = 2$
3. $x^2 - 6x + 8 = 0 \quad ; x = 4,\ x = 2$

Some example to try - Solving equations

Solve the following for x:

1. $4x - 3 = 2x - 1$
2. $\dfrac{1}{x+5} + \dfrac{1}{x-5} = \dfrac{2x+1}{x^2-25}$
3. $y = \dfrac{ax+b}{cx+d}$

Zero-product Property of Real Numbers

Zero-product Property of Real Numbers

$pq = 0$ if and only if $p = 0$ or $q = 0$ or both

Used for solving equations by factoring.

Example: Solve $(x+2)(x-5) = 0$ for x. By using the "Zero-product property" of real numbers, we should have $x+2=0$ or $x-5=0$ (or both). When $x+2=0$, $x=-2$ and when $x-5=0$, $x=5$. Therefore, we have two solutions for the given equation: $x=-2$ or $x=5$.

Extending this idea, we can solve $x(x-3)(x+5)(x-1) = 0$ for x. Then we have four possibilities: $x=0$, $x-3=0$, $x+5=0$ and $x-1=0$. Therefore, this equation has four solutions: $x=0$ or $x=3$ or $x=-5$ or $x=1$.

Example to try - Solving equations by factoring

Solve the following for x by factoring:

1. $(x-1)(x+1)=0$
2. $3x^2+2x=0$
3. $x^2+2x+1=0$
4. $x^2+5x+4=0$
5. $x^2-4x-5=0$
6. $x^2+4x-5=0$
7. $x^2=9$

Quadratic Formula and Examples to try

Quadratic Formula

The solution to a quadratic equation $ax^2 + bx + c = 0$, when $a \neq 0$ is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve the following for x:

1. $x^2 + 2x + 1 = 0$
2. $x^2 + 3x + 2 = 0$
3. $2x^2 - 5x = 0$
4. $3x^2 - 4x - 1 = 0$
5. $x^2 - 9 = 0$