MATH1550: Precalculus

Lecture 08: Section 2.2

The "OTHER" Types of Equations

In this section we discuss the following types of Equations

1. Equations involving absolute value
2. Equations involving n^{th} exponents/roots
3. Equations of "quadratic type"

We start the discussion with some unfinished discussion of absolute values and the graphical interpretation of solutions.

Graphical Interpretation of "Solutions" of an Equation: The Story of "Intersection" of Graphs

Suppose you are given an equation $p(x) = q(x)$.

Plot $y = p(x)$ and $y = q(x)$ on the same axis frame.

The solution(s) of $p(x) = q(x)$ is given by precisely the **point(s) of intersection** of the two graphs $y = p(x)$ and $y = q(x)$.

The number of points of intersection will tell us the number of **distinct solutions**.

In particular, if the two graphs $y = p(x)$ and $y = q(x)$ does not intersect, it means that the equation $p(x) = q(x)$ does not have any solutions.

On the other hand, if the two graphs $y = p(x)$ and $y = q(x)$ overlap on some interval, then the entire region will be valid solutions for the given equation.

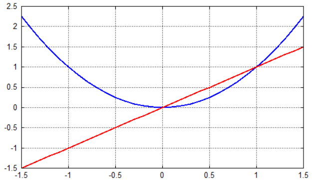
Graphical Solutions of Equations

Consider the equation $x^2 = x$. You can recognize that this can be written as the quadratic equation $x^2 - x = 0$, which can be solved by factoring: $x(x - 1) = 0$; so that the solutions are $x = 0$ or $x = 1$.

We can solve this by graphically in two ways.

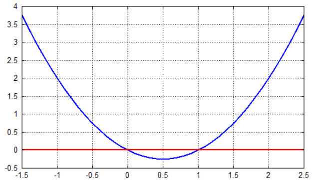
Method 1:

Plotting $y = x^2$ (in blue) and $y = x$ (in red)



Method 2:

Plotting $y = x^2 - x$ (in blue) and $y = 0$ (in red)

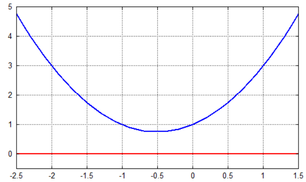


Either way, we get the same two solutions, $x = 0$ or $x = 1$.

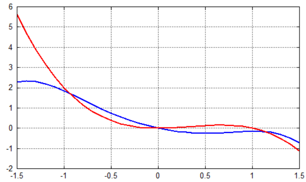
What can you say about these graphs?

Given below are 4 equations of the form $p(x) = q(x)$ which we try to solve graphically. The red color graph is $y = q(x)$, which is $y = 0$ in all except in (2). The blue color graph is $y = p(x)$.

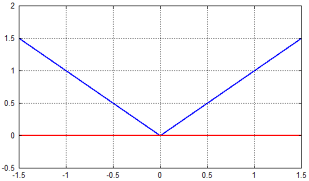
(1) $x^2 + x + 1 = 0$



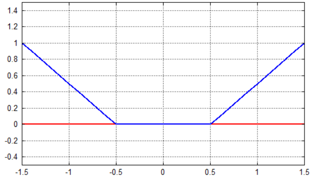
(2)



(3) $|x| = 0$



(4)



Equations Involving Absolute Value

Problem: Variable is inside absolute value sign.

Examples:

1. $|2x - 1| + 7 = 0$
2. $|x^2 - 2x| + 2x = 4$
3. $|x + 1| - |3x - 2| = 0$
4. $|x + 1||x - 2| = 1$

Two techniques:

1. Rewrite the equation removing the absolute values
   - Disadvantages: Lot of book-keeping.
   - Advantages: Does not make the equation any more completed.
2. Take the square
   - Disadvantages: Can make the equation a lot more complicated than what was started with. Introduces a lot of spurious solutions. May have to apply repeatedly.
   - Advantages: Not much of book-keeping.

Equations involving n^{th} exponents/roots

Solve the following equations. Check if all the solutions you obtained are valid.

1. $(x-2)^2 - 9 = 0$ (Hint: Use definition)
2. $(x+4)^3 + 8 = 0$ (Hint: Use definition)
3. $3x^3 - 12x^2 - 15x = 0$ (Hint: Use factorization)
4. $x^4 + x^2 - 6 = 0$ (Hint: Set $X = x^2$ and factoring/quadratic formula)
5. $x^4 - 6x^2 + 4 = 0$ (Hint: Set $X = x^2$ and factoring/quadratic formula)
6. $6x^{-2} - x^{-1} - 2 = 0$ (Hint: Set $X = x^2$ and factoring/quadratic formula)
7. $2x^{4/3} - x^{2/3} - 6 = 0$ (Hint: Set $X = x^{2/3}$ and factoring/quadratic formula)
8. $x - 2 = \sqrt{x}$ (Hint: Set $X = \sqrt{x}$ and factoring/quadratic formula)
9. $x + x^{-1} - 2 = 0$ (Hint: Multiply by x and factoring/quadratic formula)
10. $\sqrt{\frac{x-a}{x}} + 4\sqrt{\frac{x}{x-a}} = 5$ (Hint: First set $X = \sqrt{\frac{x-a}{x}}$ then multiply by X and factoring/quadratic formula)

Equations of quadratic type

Solve the following equations. Check if all the solutions you obtained are valid. In some cases, there could be **Extraneous Solutions**.

1. $\sqrt{x-3} = 5$
2. $x - 2 = \sqrt{x}$
3. $x - 2 = -\sqrt{x}$
4. $\sqrt{x-5} - \sqrt{x+4} + 1 = 0$
5. $\sqrt{x-5} - \sqrt{x+4} - 1 = 0$
6. $\sqrt{x-5} + \sqrt{x+4} + 1 = 0$
7. $\sqrt{x-5} + \sqrt{x+4} - 1 = 0$
8. $\sqrt{x+2} = x - 4$
9. $\sqrt{2x-3} - \sqrt{3x+3} + \sqrt{3x-2} = 0$
10. $\sqrt{a-x} + \sqrt{b-a} = \sqrt{a+b-2x}$ where, $b > a > 0$