MATH1550: Precalculus

Chapter 3

Functions

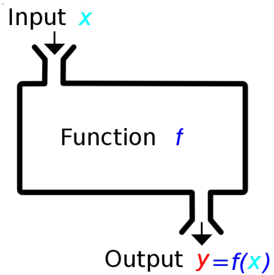
I will change the order slightly

1. Sec. 3.1: Definition of a function
2. Sec. 3.2: Graph of a function
3. Sec. 3.4: Techniques in graphing
4. Sec. 3.5: Methods of combining functions
5. Sec. 3.6: Inverse functions

   I will talk about sec. 3.3 at the end.

6. Sec. 3.3: Shapes of graphs, average rate of change

Functions



A **function** is a is a rule/process which specifies a $y-$value for each $x-$value.
The $x-$value is called the **independent variable**, and the $y-$value is called the **dependent variable**.

In other words, a **function** f is a rule/process which maps a $x-$value to a $y-$value.

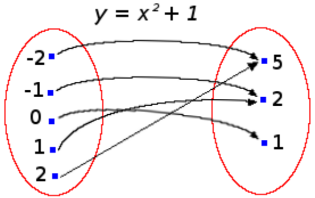
A function **should not** map a single $x-$value to many $y-$values. On the other hand, a function may map many $x-$ values to a single $y-$ value.

Representing Functions

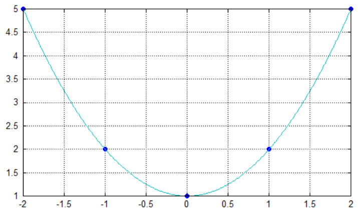
- Tables

x	$y = x^2 + 1$
-2	5
-1	2
0	1
1	2
2	5

- Set Notation

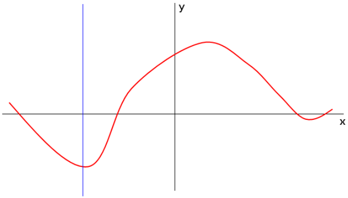


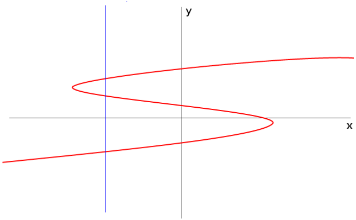
- Graphs



Vertical Line Test

Use the vertical line test to determine when the graph is known.





Domain, Range and Some Examples

- Domain:
  Values that $x-$variable (i.e. the independent variable) can take.
- Range:
  Values that $y-$variable (i.e. the dependent variable) can take.

If the values are not specified, we have to find the “maximal” values they can take.

Some examples...

Finding the Domain of a Function

We would like to have the largest domain as possible.

The values in the domain **should NOT** make the function *undefined*.

For example, if you are given a function $y = f(x)$,

- The values of x which makes the denominator, if any, of $f(x)$ zero,
- The values of x which makes the terms under the even power roots negative,

CANNOT be in the domain of function $f(x)$.

Finding the Domain: Examples

1. The domain of $f(x) = \dfrac{x}{(x-2)(x+3)}$ is all the real numbers **except** $x = -3$ and $x = 2$, because, at $x = -3$ and $x = 2$ the denominator of $f(x)$ will become zero.

2. The domain of $g(x) = x + \sqrt{x-1}$ is all the real numbers greater than or equal to 1 (i.e. $x \geq 1$). Because, if $x < 1$, the term under the square root sign will become negative.

3. The domain of $h(x) = x + x^2 + (x-2)^3$ is all the real numbers. Because, we can *evaluate* the given function at all the real numbers, without going "undefined".

4. The domain of $q(x) = \sqrt{\dfrac{(x-2)}{(x+3)}}$ is all the real numbers except those in the interval $[-3, 2)$. WHY...

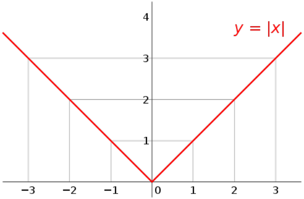
   At $x = -3$, the denominator becomes zero. Within the interval $(-3, 2)$ the term within the square root becomes negative.

Piecewise Defined Functions

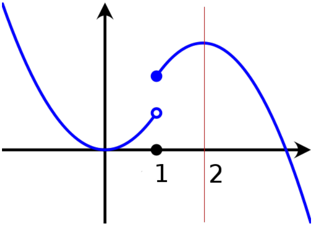
Examples:

Our good old friend, the absolute value function, $f(x) = |x|$

$$f(x) = \begin{cases} x & \text{when } x \geq 0 \\ -x & \text{when } x < 0 \end{cases}$$



$$g(x) = \begin{cases} x^2 & \text{when } x < 1 \\ -(x-2)^2 + 2 & \text{when } x \geq 1 \end{cases}$$



Translations and Reflections of Functions (§3.4)

If the graph of a function is given as $y = f(x)$

1. $y = f(x) + c$: Shift c units up
2. $y = f(x) - c$: Shift c units down
3. $y = f(x + c)$: Shift c units left
4. $y = f(x - c)$: Shift c units right
5. $y = -f(x)$: Reflection about the $x-$ axis
6. $y = f(-x)$: Reflection about the $y-$ axis

CAUTION!!! If you are asked for $y = f(-x + c)$, first do the shifting then do the reflection.

I made a mistake here in the class. I have corrected it now.

The "mathematical" word for "shift" is "translation".

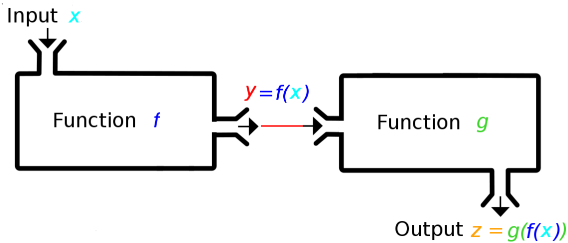
A nice online graphing calculator can be found here:

http://my.hrw.com/math06_07/nsmedia/tools/Graph_Calculator/graphCalc.html

Combining Functions (§3.5)

If we are given two functions $f(x)$ and $g(x)$, is it possible to combine them in some way? YES!

1. Combining functions arithmetically
   1. Addition of functions $(f+g)(x) = f(x) + g(x)$
   2. Subtraction of functions $(f-g)(x) = f(x) - g(x)$
   3. Multiplication of functions $(fg)(x) = (f(x)) \cdot (g(x))$
   4. Division of functions $(f/g)(x) = f(x)/g(x)$.
      BE CAREFUL !!! We can do this only when $g(x) \neq 0$
2. Composition of functions $(f \circ g)(x) = f(g(x))$

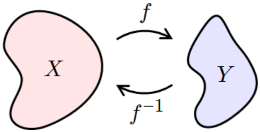


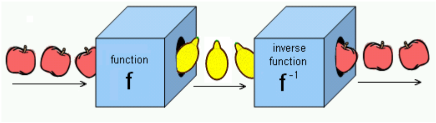
Inverse Functions (§3.6)

QUESTION: Suppose you are given a function $y = f(x)$. For each value of y, can you find a corresponding value of x?

ANSWER: May be... depending on the actual $f(x)$ given.

If we could, we have to solve the equation $y = f(x)$ for y. The solution is called the "inverse function of f", and denoted by (the rather confusing, yet very popular) notation f^{-1}.





Inverse Functions: A Simple Example

Given $y = 2x + 3$, (so, clearly $f(x) = 2x + 3$) can you find the inverse function?

We can easily solve $y = 2x + 3$ for x.

We get $x = \dfrac{y-3}{2}$, so $f^{-1}(y) = \dfrac{y-3}{2}$.

We usually write this in terms of "x" as a variable and say
$f^{-1}(x) = \dfrac{x-3}{2}$

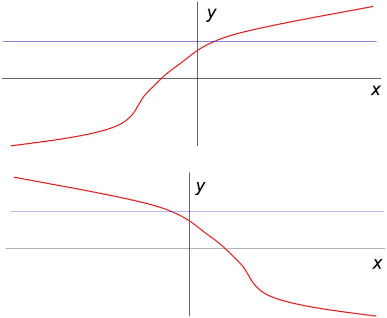
What is $f^{-1}(f(x))$? ... $\dfrac{(2x+3)-3}{2} = x$;

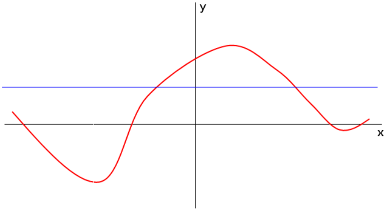
and also, $f(f^{-1}(x)) = 2\left(\dfrac{x-3}{2}\right) + 3 = x$ ...!

This is true in general.
We can use this fact to check the inverse we computed.

When can the inverse function be found???

Horizontal line test.

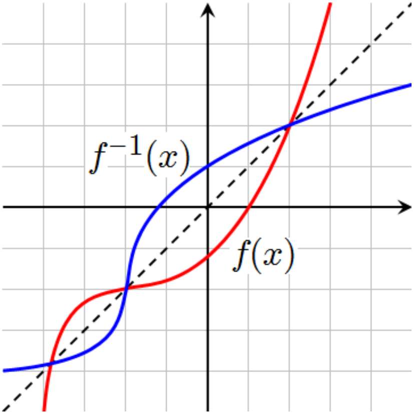




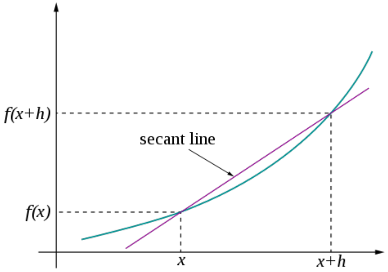
Using the "horizontal line test" we see that the function defined in the graph on right has no inverse function.

Note that, the "horizontal line test" will work if and only if the graph is increasing. In that case, for any (ONE) $x-$value there is ONE (unique) $y-$ value. This is called being **one-one** (said "one-to-one").

Inverse Functions: as the reflection over $y = x$



Average Rate of Change (§3.3)



Average Rate of change $\dfrac{\Delta y}{\Delta x} = \dfrac{f(x+h) - f(x)}{(x+h) - x}$

Δy (said "delta y") denotes the change along the $y-$axis.
Similarly, Δx (said "delta x") denotes the change along the $x-$axis.

The denominator can be simplified : $x + h - x = x$

This formula is known as the "difference quotient".

The *average rate of change* gives an "approximation" of the slope for a graph/function which is not a line.

Bit of Terminology...

x-intercept: point where the graph crosses the x-axis.

Found by setting $y = 0$ and solving for x.

y-intercept: point where the graph crosses the y-axis.

Found by setting $x = 0$ and solving for y.

maximum value: the range value that is greater than all other function values and the domain value that gives it.

Found by graphical observation.

minimum value: the range value that is less than all other function values and the domain value that gives it.

Found by graphical observation.

increasing: a function is increasing if for $a < b$, $f(a) < f(b)$.

Found by graphical observation.

decreasing: a function is decreasing if for $a < b$, $f(a) > f(b)$.

Found by graphical observation.

turning point: the point where a function changes from increasing to decreasing or decreasing to increasing.

Found by graphical observation.

Quick Exercise