MATH1550: Precalculus

Supplementary Notes

Translations and Reflections of Functions (§3.4)

If the graph of a function is given as $y = f(x)$

1. $y = f(x) + c$: Shift c units up
2. $y = f(x) - c$: Shift c units down
3. $y = f(x + c)$: Shift c units left
4. $y = f(x - c)$: Shift c units right
5. $y = -f(x)$: Reflection about the $x-$ axis
6. $y = f(-x)$: Reflection about the $y-$ axis

CAUTION!!! If you are asked for $y = f(-x + c)$, first do the shifting then do the reflection.

I made a mistake here in the class. I have corrected it now.

The "mathematical" word for "shift" is "translation".

A nice online graphing calculator can be found here:

`http://my.hrw.com/math06_07/nsmedia/tools/Graph_Calculator/graphCalc.html`

SIX step process

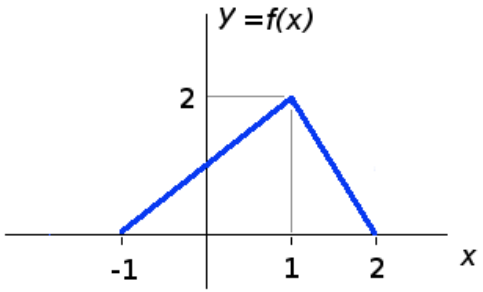
Given $y = f(x)$, and asked to sketch $y = af(bx + c) + d$.

1. Identify the key points.
2. Find the **inverse** of the term in parenthesis (i.e. term in parenthesis is $bx + c$, hence its inverse is $(x - c)/b$). This inverse function will define how the "new" x will behave, compared to the "old" x.

   This is VERY important, but I think I did not do this - SORRY
3. The "new" y will be given by simply $ay + d$.
4. Now find how the key points are mapped using the inverse function for x and function for y.
5. Plot the mapped key points on a graph.
6. Complete the graph using the knowledge of the original graph.

Example

Given $y = f(x)$ in the graph, and asked to sketch $y = 2f(-3x + 2)$.



1. Identify the key points.

   Key points are: $(-1, 0)$, $(1, 2)$ and $(2, 0)$

2. Find the **inverse** of the term in parenthesis (i.e. term in parenthesis is $bx + c$, hence its inverse is $(x - c)/b$).

   Inverse of the parenthesized term, $-3x + 2$ is clearly

   $$new\ X = \frac{(x - 2)}{(-3)}$$

3. The "new" y will be given by simply $ay + d$.

   $$new\ Y = 2y$$

continued ...

Example *continued...*

4. Now find how the key points are mapped using the inverse function for x and function for y.

Old x	Old y	New $x = \dfrac{x-2}{-3}$	New $y = 2y$
-1	0	$\dfrac{-1-2}{-3} = 1$	$(2)(0) = 0$
1	2	$\dfrac{1-2}{-3} = \dfrac{1}{3}$	$(2)(2) = 4$
2	0	$\dfrac{2-2}{-3} = 0$	$(2)(0) = 0$

5. Plot the mapped key points on a graph.

6. Complete the graph using the knowledge of the original graph.