MATH1550: Precalculus

Chapter 5

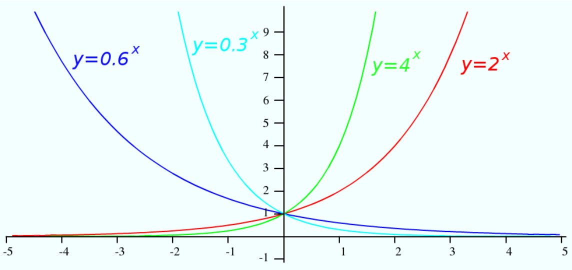
Exponential Functions (§5.1)

Exponential Function with Base b

$y = b^x$ for $b > 0$ and $b \neq 1$

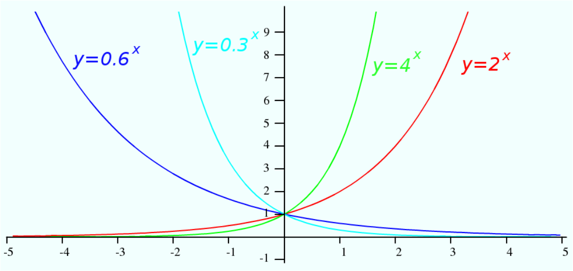
Properties of Exponential Functions

- If $b^x = b^y$ then $x = y$ as long as $b \neq 1$.
- If $b > 1$ the function is increasing.
  The larger b is, the faster the function increases.
- If $0 < b < 1$ the function is decreasing.
  The smaller b is, the faster the function decreases.



Graphs of Exponential Functions $y = b^x$

- has no $x-$intercept
- $y-$intercept is 1
- has a horizontal asymptote $y = 0$ depending on $b > 1$ or $b < 1$
- Domain: all real numbers
- Range: POSITIVE real numbers



Mathematically the most useful base is ... e

$e \approx 2.718281828459045235 36\ldots$

This is an irrational number (like π, $\sqrt{2}$ etc.)

§5.2 talks more about this number e. Please read this section.

L.T.S. = Left To Students

Graphing Exponential Functions

Use shifting, reflection and scaling to sketch the following functions

1. $y = -3^x + 3$
2. $y = 4^{x+1} - 1$
3. $y = -2\left(2^{-x+1}\right) + 2$

Logarithmic Function (§5.3)

Logarithmic Function with base b

Informally, it is defined as the "inverse function" of the exponential function.

Given $x = b^y$, we write $y = \log_b x$,

(said "logaritm to/in base b of x" or simply "log b of x")

If the base is e ($\approx 2.71828\ldots$), it is called the **natural logarithm**.

"$\log_e x$" is often written "$\ln x$"

Exercises

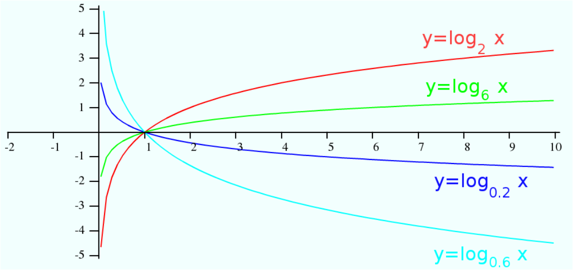
1. Write in the logarithm form
   1. $\clubsuit^{\spadesuit} = \diamondsuit$
   2. $2^{-5} = \frac{1}{32}$
   3. $3^4 = 81$
   4. $e^{4t} = 16$
2. Write in the exponential form
   1. $\log_{\clubsuit} \diamondsuit = \spadesuit$
   2. $\ln z = 19$
   3. $\log_4 2 = 16$

More Exercises

1. Evaluate
   1. $\log_2 8$
   2. $\log_{25} 1$
   3. $\ln e$
   4. $\log_5 5\sqrt{5}$
   5. $\log_4 \frac{1}{16}$
   6. $\log_b b^a$
2. Solve
   1. $\log_5 x = 2$
   2. $\ln x = -e$
   3. $\log_4 16 = x$

Graphs of Exponential Functions $y = b^x$

- has no $y-$intercept
- $x-$intercept is 1
- has a vertical asymptote $x = 0$
- Domain: POSITIVE real numbers
- Range: All real numbers



Graphing Exponential Functions

Use shifting, reflection and scaling to sketch the following functions

1. $y = \log_3(x - 2) + 1$
2. $y = \ln(x + e)$
3. $y = -\log_{10}(x + 1)$
4. $y = \log_e(-x)$

Properties of Logarithms §5.4

- When the base is the same as the argument the logarithm equals 1. $\log_b b = 1$
- The logarithm of 1 is always 0. $\log_b 1 = 0$
- When a base is raises to a logarithm with the same base the expression equals the argument.
  $b^{\log_b x} = x$
- The logarithm of a product is equal to the sum of the logarithms of the factors.
  $\log_b(MN) = \log_b M + \log_b N$
- The logarithm of a quotient is the equal to the logarithm of the numerator minus the logarithm of the denominator.
  $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$
- The logarithm of an argument with an exponent is equal to the exponent times the logarithm of the argument without the exponent.
  $\log_b x^n = n \log_b x$
- The base of logarithm can be changed by dividing the logarithm with the wanted of the argument by the logarithm with the wanted base of the old base.
  $\log_a x = \frac{\log_b x}{\log_b a}$

Exercises

1. Simplify to one logarithm
   1. $\log_2 9 + \log_2 \left(\frac{5}{18}\right) - \log_2 \left(\frac{5}{2}\right)$
   2. $3 \ln 2 - \ln 16 + \frac{1}{3} \ln 8$
2. Expand the following
   1. $\log_b \dfrac{\sqrt{1-x^2}}{x}$
   2. $\ln \dfrac{x^2}{\sqrt{1+x^2}}$
   3. $\log_b \sqrt[3]{\dfrac{x+3}{x}}$
3. Solve the following equations. Express your answer in terms of **natural logarithms**
   1. $2^x = 13$
   2. $3e^{1+t} = 2$
   3. $5^{3x-1} = 27$

Equations and Inequalities with Logarithms (§5.5)

Never take the logarithm/exponential of both sides

Rewrite in exponential or logarithmic form!

Look out for EXTRANEOUS ROOTS!

Solve the following equations

1. $7^{-4x} = 2^{1+3x}$
2. $\log_2(2x^2 - 4) = 5$
3. $\ln 4 - \ln x = \dfrac{\ln 4}{\ln x}$
4. $\log_6 x + \log_6(x + 1) = 1$
5. $\log_2(2x^2 + 4) = 5$
6. $\log_2(x + a) - \log_2(ax) = 1$

Equations and Inequalities with Logarithms

Preserving the inequality

- $p < q \Leftrightarrow b^p < b^q$
- $p < q \Leftrightarrow \log_b p < \log_b q$

Solve the following equations

1. $2\left(1 + 0.4^x\right) < 5$
2. $\ln(2 - 3x) \leq 1$
3. $e^{2-3x} \leq 1$
4. $\log_{10} x + \log_{10}(x + 2) \leq \log_{10} 24$

Compound Interest (§5.6)

Compound Interest Formula

Interest that is applied to the balance of an account at the end of a compounding period.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

- $A =$ **amount:** the amount in the account after interest is added
- $P =$ **principal:** the amount in the account before interest is added
- $r =$ **interest rate:** the interest rate *in decimal form*
- $n =$ **number of compoundings per year:**
- $t =$ **time:** the number of years

Interest Compoundings

- annually : n=1
- semiannually : n=2
- quarterly : n=4
- monthly : n=12
- weekly : n=52
- daily : n=365

Continuously Compounded Interest

Continuously Compound Interest Formula

Interest that is constantly being compounded.

$$A = Pe^{rt}$$

- A = **amount:** the amount in the account after interest is added
- P = **principal:** the amount in the account before interest is added
- r = **interest rate:** the interest rate *in decimal form*
- e = **natural base:** 2.71828...
- t = **time:** the number of years

Exponential Growth and Decay (§5.7)

Exponential Growth and Decay Formula

Interest that is constantly being compounded.

$$\mathcal{N}(t) = \mathcal{N}_0 e^{kt}$$

- $\mathcal{N}(t)$= function notation for the size of a population at a given time
- $\mathcal{N}_0$= the initial population
- **e=natural base:** 2.71828...
- **k=growth or decay constant:** the rate of growth or decay
- **t=time:** use the given units