MATH1550: Precalculus

Supplementary Notes

Order of Precedence and Factoring

Order of Precedence in Mathematics

Start from the inner most level and work the way out

1. Parenthesis/Brackets: $(\ldots)$, $[\ldots]$, $\{\ldots\}$
2. Powers and roots
3. Multiplication and division
4. Addition and subtraction

Example:

$$\begin{aligned}
3+\sqrt{2+(2\times 3^2-4)} &= 3+\sqrt{2+(2\times 9-4)} \\
&= 3+\sqrt{2+(18-4)} \\
&= 3+\sqrt{2+14} \\
&= 3+\sqrt{16} \\
&= 3+4 \\
&= 7
\end{aligned}$$

The need of a precedence order

Consider the expression $2 \times 3 + 4$

If we did not have a standard precedence order, we can interpret this statement in two ways:

1. $(2 \times 3) + 4 = 6 + 4 = 10$ ACCEPTED
2. $2 \times (3 + 4) = 2 \times 7 = 14$ UNACCEPTED

This is ambiguous. Therefore, we have "agreed" that the first one is correct.
(*Well, philosophically, either one is fine but due to many other reasons, the first one is the accepted standard*)

Factoring

Factoring is the process of "pulling out" (**factoring**) multiplicative components (**factors**) from a mathematical expression, without changing the value of the expression.

Example: given $ax^2 + a^2x$, we can factor out ax and write $ax^2 + a^2x = ax(a + x)$.

If we want, we can even factor out a^2x^2 and write $ax^2 + a^2x = a^2x^2(\frac{1}{a} + \frac{1}{x})$.

To see if the factoring is correct, we can multiply out and see if we get the original expression back!

Recognizing the possible factors is an essential skill in factoring. Often, you are not told what to factor out; you should be able to what could be factored out from a given expression.

Try these yourself

1. Factor out x from $x^2 + 2x$
2. Factor out xyz from $x^2yx + xy^2z + xyz^2$
3. Factor out xy from $x + y$
4. Factor out $x + 2$ from $3x(x + 2) + 4(x + 2)$
   (Hint: to avoid confusion, think $(x + 2)$)
5. Factor out $x + y$ from $x(x + y)z + (x + y)yz + xy(x + y)$

Some important factoring rules

1. Factoring the "difference of square"
$$a^2 - b^2 = (a-b)(a+b)$$
2. Factoring the "difference of cubes"
$$a^3 - b^3 = (a-b)(a^2+ab+b^2)$$
3. Factoring the "sum of cubes"
$$a^3 + b^3 = (a+b)(a^2-ab+b^2)$$

Can you verify them?

Now try these...

1. $x^2 - \frac{1}{4}$; (difference of squares)
2. $y^{12} - x^6$; (difference of squares)
3. $y^{12} - x^6$; (difference of cubes)
4. $y^{12} + x^6$; (sum of cubes)

Factoring Quadratic Polynomials

Given a quadratic polynomial

1. Rearrange to the standard form $ax^2 + bx + c$
   (To make the process easier, make sure that a is positive. If not factor out (-1) from the whole expression.)
2. Multiply a and c
3. List the pairs of factors of $a \times c$
4. Select the pair of factors of $a \times c$ which add up to b; call them b_1 and b_2
5. Rewrite in the form of $ax^2 + b_1x + b_2x + c$
6. Then, (possibly after interchanging b_2 and b_2) factor by grouping.

Factoring Quadratic Polynomials: An Example

Given a quadratic polynomial $8 + 3x^2 + 10x$

1. Rearrange to the standard form $ax^2 + bx + c$
   $3x^2 + 10x + 8$; and $a = 3,\ b = 10,\ c = 8$; a positive
2. Multiply a and c
   $a \times c = 3 \times 8 = 24$
3. List the pairs of factors of $a \times c$
   $24 = 1 \times 24 = 2 \times 12 = 3 \times 8 = 4 \times 6$
4. Select the pair of factors of $a \times c$ which add up to b; call them b_1 and b_2
   $1 + 24 = 25;\ \ 2 + 12 = 14;\ \ 3 + 8 = 11;\ \ 4 + 6 = 10$
   Therefore, take $b_1 = 4$ and $b_2 = 6$
5. Rewrite in the form of $ax^2 + b_1x + b_2x + c$
   $3x^2 + 4x + 6x + 8$
6. Then, (possibly after interchanging b_2 and b_2) factor by grouping.
   Factor x from the first two terms and 2 from the last two
   $x(3x + 4) + 2(3x + 4)$. Now we see that $3x + 2$ is a common factor, so, factor it out: $(x + 2)(3x + 4)$

Factoring Quadratic Polynomials: Another Example

Given a quadratic polynomial $x^2 - 5x + 6$

1. Rearrange to the standard form $ax^2 + bx + c$
   $x^2 - 5x + 6$; and $a = 1,\ b = -5,\ c = 6$; a positive
2. Multiply a and c
   $a \times c = 1 \times 6 = 6$
3. List the pairs of factors of $a \times c$
   Since b is negative **and** a and c positive, its sufficient to look only at negative factors (WHY?: you'll see the reason soon ...)
   $6 = 1 \times 6 = 2 \times 3 = (-1) \times (-6) = (-2) \times (-3)$
4. Select the pair of factors of $a \times c$ which add up to b; call them b_1 and b_2
   $1 + 6 = 7;\ \ 2 + 3 = 5;\ \ (-1) + (-6) = (-7);\ \ (-2) + (-3) = (-5)$
   Therefore, take $b_1 = -2$ and $b_2 = -3$
5. Rewrite in the form of $ax^2 + b_1x + b_2x + c$
   $x^2 + (-2)x + (-3)x + 6$
6. Then, (possibly after interchanging b_2 and b_2) factor by grouping.
   Factor x from the first two terms and 3 from the last two
   $x(x + (-2)) + 3((-1)x + 2) = x(x - 2) + 3(-x + 2)$. Almost there! I am going to factor out a (-1) from the second parenthesis:
   $x(x - 2) + (-1)3(x + (-1)2) = x(x - 2) - 3(x - 2)$ Now we see that $x - 2$ is a common factor, so, factor it out: $(x - 2)(x - 3)$

Factoring Quadratic Polynomials: Yet Another Example

Given a quadratic polynomial $x^2 + x - 6$

1. Rearrange to the standard form $ax^2 + bx + c$
   $x^2 + x - 6$; and $a = 1$, $b = 1$, $c = -6$; a positive
2. Multiply a and c
   $a \times c = 1 \times (-6) = -6$
3. List the pairs of factors of $a \times c$
   $-6 = 1 \times (-6) = (-1) \times 6 = (-2) \times 3 = 2 \times (-3)$
4. Select the pair of factors of $a \times c$ which add up to b; call them b_1 and b_2
   $1 + (-6) = (-5)$; $(-1) + 6 = 5$; $(-2) + 3 = 1$; $2 + (-3) = (-1)$
   Therefore, take $b_1 = -2$ and $b_2 = 3$
5. Rewrite in the form of $ax^2 + b_1x + b_2x + c$
   $x^2 + (-2)x + 3x - 6$
6. Then, (possibly after interchanging b_2 and b_2) factor by grouping.
   Factor x from the first two terms and 3 from the last two
   $x(x + (-2)) + 3(x - 2) = x(x - 2) + 3(x - 2)$. Now we see that $x - 2$ is a common factor, so, factor it out: $(x - 2)(x + 3)$

(Note that if we were given $x^2 - x - 6$, it would be factored as $(x + 2)(x - 3)$.
The details are right on this slide! Can you see them?)

Factoring Quadratic Polynomials: A few more Examples

- To factor out $-3x^2 - 10x - 8$, first factor out (-1). (This step is not necessary, but would help avoid several sign related errors.) Then we get $(-1)(3x^2 + 10x + 8)$. We have already factored out $3x^2 + 10x + 8 = (x + 2)(3x + 4)$. Therefore, we can write $-3x^2 - 10x - 8 = (-1)(x + 2)(3x + 4)$
- Same story with $-x^2 + 5x - 6$ and $-x^2 + x + 6$.

Try it yourself

1. $x^2 + 5x + 6$
2. $x^2 - 3x + 2$
3. $6x^2 - x - 1$
4. $-6x^2 - x + 2$
5. $x^2 + 2x + 1$

Answers

1. $x^2 + 5x + 6 = (x + 2)(x + 3)$
2. $x^2 - 3x + 2 = (x - 1)(x - 2)$
3. $6x^2 - x - 1 = (3x + 1)(2x - 1)$
4. $-6x^2 - x + 2 = (-1)(3x + 2)(2x - 1)$
5. $x^2 + 2x + 1 = (x + 1)(x + 1)$