A small note:
Note that there is a subtle difference between the following questions:

1. Evaluate $\sin^{-1}\left(\frac{1}{2}\right)$
2. Find θ such that $\sin\theta = \frac{1}{2}$ (usually the expected quadrant will be given)
3. Solve $\sin\theta = \frac{1}{2}$ (with NO quadrant specification is given)

The "expected" answers are as follows (in the same order):

1. $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ (or $30°$); from the definition of the $sin - inverse$ function.
2. Find θ such that $\sin\theta = \frac{1}{2}$: Here θ can be in first or third quadrant and so, $\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ (i.e. $30°$ or $150°$). The "solution" has to be picked based on the quadrant asked for.

   The quadrant can be specified in may ways, e.g. specifying the sign ($+$ or $-$) of another trig. function, the values the angles can take, or directly saying the quadrant expected.
3. Solve $\sin\theta = \frac{1}{2}$. For this one we can have "*any*" angle θ which will give $\sin\theta = \frac{1}{2}$, so we have to consider *not only* the possible quadrants, but also add (and subtract) full circles (i.e. integer multiples of either 2π radians or $360°$).

   So, the solution set will be $\theta = \frac{\pi}{6} + 2\pi k$ or $\frac{5\pi}{6} + 2\pi k$, where $k = \cdots -3, -2, -1, 0, 1, 2, 3, \ldots$. If you prefer degrees, $\theta = 30° + k360°$ or $150° + k360°$, where $k = \cdots -3, -2, -1, 0, 1, 2, 3, \ldots$.

Summary of domains and ranges for the inverse trig functions

Function	Domain	Range
$\sin^{-1}$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\cos^{-1}$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}$	$(-\infty, \infty)$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\cot^{-1}$	$(-\infty, \infty)$	$[0, \pi]$
$\csc^{-1}$	$(-\infty, -1] \cup [1, \infty)$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\sec^{-1}$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi]$

Worked examples (Solutions given at the end)

Problems:

1. Using the domain and range for evaluating inverse trig functions
   (a) $\sin^{-1}\left(\frac{1}{2}\right)$ (b) $\sin^{-1}\left(-\frac{1}{2}\right)$ (c) $\cos^{-1}\left(\frac{1}{2}\right)$ (d) $\cos^{-1}\left(-\frac{1}{2}\right)$
2. Some **"fun"** problems using trig & inverse trig functions: Evaluate the following
   (a) $\sin\left(\sin^{-1}(x)\right)$ (b) $\dfrac{1}{\cot\left(\tan^{-1}(x)\right)}$ (c) $\sin\left(\frac{\pi}{2} - \cos^{-1}(x)\right)$ (d) $\cos^{-1}(2)$
3. Some more **"fun"** problems using trig & inverse trig functions: Evaluate the following
   (a) $\sin\left(\cos^{-1}\left(\frac{5}{13}\right)\right)$ (b) $\sec\left(\tan^{-1}(-4)\right)$ (c) $\sin\left(2\cos^{-1}\left(\frac{3}{5}\right)\right)$ (d) $\cos\left(\cot^{-1}(3)\right)$

Exercises: Evaluate/Solve

(i) $\tan^{-1}\left(\sqrt{3}\right)$ (ii) $\cot^{-1}\left(-\sqrt{3}\right)$ (iii) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ (iv) $\sec^{-1}(0.1)$ (v) $\tan\left(\sec^{-1}(2)\right)$
(vi) $\tan\left(\csc^{-1}(2)\right)$ (vii) $\cos\left(2\sin^{-1}\left(\frac{1}{3}\right)\right)$ (viii) $\cos\left(-\cos^{-1}\left(\frac{3}{4}\right)\right)$ (ix) $\sin\left(\tan^{-1}\left(\frac{1}{5}\right)\right)$
(x) Solve $\sin\theta = \frac{\sqrt{3}}{2}$, for θ. (xi) Solve $\tan\theta = \sqrt{3}$, for θ. (xii) Solve $\sec\theta = 2$, for θ.

Solutions:

1. Using the domain and range for evaluating inverse trig functions

   (a) $\sin^{-1}\left(\frac{1}{2}\right) = \dfrac{\pi}{6}$; (b) $\sin^{-1}\left(-\frac{1}{2}\right) = -\dfrac{\pi}{6}$;

   (c) $\cos^{-1}\left(\frac{1}{2}\right) = \dfrac{\pi}{3}$; (d) $\cos^{-1}\left(-\frac{1}{2}\right) = \dfrac{2\pi}{3}$;

2. Some **"fun"** problems using trig & inverse trig functions: Evaluate the following

   (a) $\sin\left(\sin^{-1}(x)\right) = x$

   (b) $\dfrac{1}{\cot\left(\tan^{-1}(x)\right)} = \tan\left(\tan^{-1}(x)\right) = x$

   (c) $\sin\left(\frac{\pi}{2} - \cos^{-1}(x)\right) = \cos\left(\cos^{-1}(x)\right) = x$

   (d) Since 2 is NOT in the domain of of $\cos^{-1}$ function, $\cos^{-1}(2)$ HAS NO SOLUTION !!!

3. Some more **"fun"** problems using trig & inverse trig functions: Evaluate the following

   (a) $\sin\left(\cos^{-1}\left(\frac{5}{13}\right)\right)$

   We can use either the Pythagorean identity ($\sin^2\theta + \cos^2\theta = 1$) or the unit circle to solve this. The calculations involved are the same (more or less)

   I will use the Pythagorean identity. For the purpose of substituting in to the Pythagorean formula, we have to take $\theta = \cos^{-1}\left(\frac{5}{13}\right)$.

   Then we get, $\sin^2\left(\cos^{-1}\left(\frac{5}{13}\right)\right) + \cos^2\left(\cos^{-1}\left(\frac{5}{13}\right)\right) = 1$.

   This stands for $\left(\sin\left(\cos^{-1}\left(\frac{5}{13}\right)\right)\right)^2 + \left(\cos\left(\cos^{-1}\left(\frac{5}{13}\right)\right)\right)^2 = 1$.

   We know that $\cos\left(\cos^{-1}\left(\frac{5}{13}\right)\right) = \frac{5}{13}$.

   So, we get, $\left(\sin\left(\cos^{-1}\left(\frac{5}{13}\right)\right)\right)^2 + \left(\frac{5}{13}\right)^2 = 1$.

   After simplifying a little bit we get, $\left(\sin\left(\cos^{-1}\left(\frac{5}{13}\right)\right)\right)^2 = \left(\frac{12}{13}\right)^2$.

   Which means $\sin\left(\cos^{-1}\left(\frac{5}{13}\right)\right) = \pm\frac{12}{13}$.

   However, $\cos^{-1}\left(\frac{5}{13}\right)$ gives an angle in the first quadrant.

   Therefore, the final answer is $\boxed{\sin\left(\cos^{-1}\left(\frac{5}{13}\right)\right) = \frac{12}{13}}$

   (b) $\sec\left(\tan^{-1}(-4)\right)$

   This is same as the earlier one, if we use the Pythagorean identity $\tan^2\theta + 1 = \sec^2\theta$.

   Start by setting $\theta = \tan^{-1}(-4)$ to get $\tan^2\left(\tan^{-1}(-4)\right) + 1 = \sec^2\left(\tan^{-1}(-4)\right)$.

   Which is $\left(\tan\left(\tan^{-1}(-4)\right)\right)^2 + 1 = \left(\sec\left(\tan^{-1}(-4)\right)\right)^2$.

   Using the fact that $\tan\left(\tan^{-1}(-4)\right) = -4$ and simplifying, we get, $\left(\sec\left(\tan^{-1}(-4)\right)\right)^2 = 17$.
   So that $\sec\left(\tan^{-1}(-4)\right) = \pm\sqrt{17}$.

   Since $\tan^{-1}(-4)$ is in the 4^{th} quadrant, and sec (which is the reciprocal of cos) is positive in the 4^{th} quadrant, see that, $\boxed{\sec\left(\tan^{-1}(-4)\right) = \sqrt{17}}$.

(c) $\sin\left(2\cos^{-1}\left(\frac{3}{5}\right)\right)$

First of all, set $\theta = \cos^{-1}\left(\frac{3}{5}\right)$. So, we are basically asked to find $\sin 2\theta$.

Using the "double angle" formula, $\sin 2\theta = 2\sin\theta\cos\theta$.

Hence, $\sin\left(2\cos^{-1}\left(\frac{3}{5}\right)\right) = 2\sin\left(\cos^{-1}\left(\frac{3}{5}\right)\right)\cos\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$.

Clearly, $\cos\left(\cos^{-1}\left(\frac{3}{5}\right)\right) = \frac{3}{5}$.

So the "challenge" now is to find $\sin\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$.
Which can be easily done using the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$.

Just like in part (a), we get *[I leave it for you to complete the details]* $\sin\left(\cos^{-1}\left(\frac{3}{5}\right)\right) = \frac{4}{5}$.

Therefore, $\sin\left(2\cos^{-1}\left(\frac{3}{5}\right)\right) = 2\sin\left(\cos^{-1}\left(\frac{3}{5}\right)\right)\cos\left(\cos^{-1}\left(\frac{3}{5}\right)\right) = (2)\left(\frac{4}{5}\right)\left(\frac{3}{5}\right) = \frac{24}{25}$.

$$\boxed{\sin\left(2\cos^{-1}\left(\frac{3}{5}\right)\right) = \frac{24}{25}}$$

(d) $\cos\left(\cot^{-1}(3)\right)$

We may have to use the Pythagorean twice if we ant to do it that way. On the other hand, if we use the unit circle, we can do it at once.

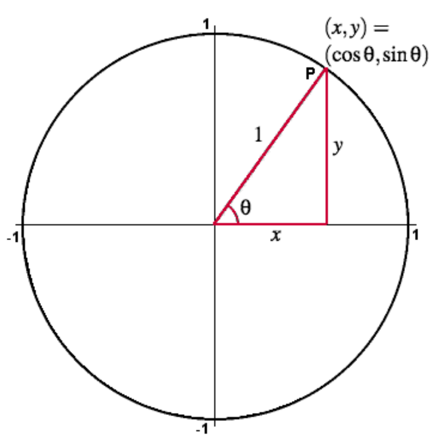
Using the Pythagorean (twice ...)

First we can use the $1 + \cot^2\theta = \csc^2\theta$ to first find $\csc\left(\cot^{-1}(3)\right)$. If you do the calculation, it can be seen that $\csc\left(\cot^{-1}(3)\right) = \sqrt{10}$. So, we get $\sin\left(\cot^{-1}(3)\right) = \frac{1}{\sqrt{10}}$ from the reciprocal identity.

We are not done yet, we have to use the Pythagorean $\sin^2\theta + \cos^2\theta = 1$ to find $\cos\left(\cot^{-1}(3)\right)$ using the result for $\sin\left(\cot^{-1}(3)\right)$.

Eventually, we will get that $\cos\left(\cot^{-1}(3)\right) = \frac{3}{\sqrt{10}}$.

Using unit circle



First set $\theta = \cot^{-1}(3)$.

Also recall that $\cot\theta = \frac{x}{y}$.

So, we may write, $\theta = \cot^{-1}\left(\frac{x}{y}\right)$.

Hence we see that $\frac{x}{y} = 3$, and this tells us that $x = 3y$.

From the unit circle, we know that $x^2 + y^2 = 1$.

Substituting for x in this equation, $(3y)^2 + y^2 = 1$.

Which gets simplified to $9y^2 + y^2 = 1$; which is $10y^2 = 1$.

So, solving for y, we get $y = \pm\frac{1}{\sqrt{10}}$. But we know that $\cot^{-1}(3)$ is in the first quadrant, so $y = \frac{1}{\sqrt{10}}$. Hence $x = \frac{3}{\sqrt{10}}$.

Since $\cos\theta = x$ on the unit circle, will get that $\boxed{\cos\left(\cot^{-1}(3)\right) = \frac{3}{\sqrt{10}}}$, as before.