Pop Quiz

1. Given $p(x) = 3x^2 + 4x$, find $p(x + 2)$.
2. Simplify
   (a) $\sqrt[3]{\dfrac{-8b^4}{a^3b^{10}}}$
   (b) $\sqrt{\dfrac{(-2)^2(x+1)^4}{(x+2)^2}}$

Answers

1. Given $p(x) = 3x^2 + 4x$, find $p(x+2)$.

$$\begin{aligned} p(x) &= 3x^3 + 4x \\ p(x+2) &= 3(x+2)^2 + 4(x+2) \\ &= 3(x+2)(x+2) + 4(x+2) \\ &= 3(x(x+2) + 2(x+2)) + 4x + 8 \\ &= 3(x^2 + 2x + 2x + 4) + 4x + 8 \\ &= 3(x^2 + 4x + 4) + 4x + 8 \\ &= 3x^2 + 12x + 12 + 4x + 8 \\ p(x+2) &= 3x^2 + 16x + 20 \end{aligned}$$

$$\boxed{p(x+2) = 3x^2 + 16x + 20}$$

Comments: This was, in general correctly done by almost all.

2. Simplify

   (a) $\sqrt[3]{\dfrac{-8b^4}{a^3b^{10}}}$

$$\begin{aligned} \sqrt[3]{\frac{-8b^4}{a^3b^{10}}} &= \sqrt[3]{\frac{(-2)^3b^4}{a^3b^{10}}} \\ &= \sqrt[3]{\frac{(-2)^3b^4b^{-10}}{a^3}} \\ &= \sqrt[3]{\frac{(-2)^3b^{(4-10)}}{a^3}} \\ &= \sqrt[3]{\frac{(-2)^3b^{-6}}{a^3}} \\ &= \left(\frac{(-2)^3b^{-6}}{a^3}\right)^{1/3} \\ &= \frac{((-2)^3)^{1/3}(b^{-6})^{1/3}}{(a^3)^{1/3}} \\ &= \frac{(-2)b^{-2}}{a} \\ \sqrt[3]{\frac{-8b^4}{a^3b^{10}}} &= \frac{-2}{ab^2} \end{aligned}$$

$$\boxed{\sqrt[3]{\frac{-8b^4}{a^3b^{10}}} = \frac{-2}{ab^2}}$$

Comments: This was, in general correctly done by many.

(b) $\sqrt{\dfrac{(-2)^2(x+1)^4}{(x+2)^2}}$

You have to be very careful with this problem. First, the -2 is an obvious danger! Then the not so obvious, yet even more critical, is the way you handle the terms containing the variable variable x.
For example, suppose $x = -1.5$. then you end up with $\sqrt{(-2)^2(-0.5)^4/(0.5)^2} = \sqrt{(4)(0.5^2)} = 1$, which is the correct answer. Whereas, if you just "canceled" the exponents with the roots, you would get (the incorrect answer) $(-2)(-0.5)^2/(0.5) = -1$ (provided no more mistakes are made).

$$\begin{aligned}
\sqrt{\frac{(-2)^2(x+1)^4}{(x+2)^2}} &= \sqrt{\frac{4(x+1)^4}{(x+2)^2}} \\
&= \sqrt{\frac{4(x+1)^4}{(x+2)^2}} \\
&= \sqrt{\frac{4((x+1)^2)^2}{(x+2)^2}} \\
&= \frac{2|(x+1)^2|}{|x+2|} \\
\sqrt{\frac{(-2)^2(x+1)^4}{(x+2)^2}} &= \frac{2(x+1)^2}{|x+2|} \qquad \textit{Since } |x^2| = x^2
\end{aligned}$$

$$\boxed{\sqrt{\frac{(-2)^2(x+1)^4}{(x+2)^2}} = \frac{2(x+1)^2}{|x+2|}}$$

Comments: This was, in general NOT correctly done by many. This was a trick question, intended to test some common mistakes.