

## 1. Functions

- Definition: As a rule which maps an input (*independent variable*) to an output (*dependent variable*).
- Representation: Graphs, Tables, Set notation (pictorial view)
- Vertical line test ...
- Domain: All the values that can the independent variable can take
- Range: All the values that can the dependent variable takes
- Piecewise defined functions ...

## 2. Translations and Reflections of Functions

- $y = f(x) + c$  : Shift  $c$  units up
- $y = f(x) - c$  : Shift  $c$  units down
- $y = f(x + c)$  : Shift  $c$  units left
- $y = f(x - c)$  : Shift  $c$  units right
- $y = -f(x)$  : Reflection about the  $x$ - axis
- $y = f(-x)$  : Reflection about the  $y$ - axis

If you are asked for  $y = f(-x + c)$ , first do the shifting then do the reflection.

## 3. Combining Functions

- Horizontal line test ...
- Finding the inverse: Given  $y = f(x)$ , solve for  $x$ , in terms of  $y$ , or, replace  $x$  with  $y$  in  $y = f(x)$  and solve for  $y$ , in terms of  $x$ .
- Inverse is the reflection about  $y = x$ .

## 4. Average Rate of Change

$$\frac{\Delta y}{\Delta x} = \frac{f(t) - f(a)}{t - a}$$

## 5. Terminology

- x-intercept:** ...
- y-intercept:** ...
- maximum value:** ...
- minimum value:** ...
- increasing:** ...
- decreasing:** ...
- turning point:** ...

## 6. Quadratic Functions

### 7. Completing the Square ...

$$f(x) = ax^2 + bx + c$$

Factor out the coefficient of  $x^2$ , (i.e.  $a$ ), then add and subtract square of one half of the coefficient of the  $x$  term (i.e.  $(b/2a)$ )

⋮

$$f(x) = a \left( x + \left( \frac{b}{2a} \right) \right)^2 + \left( \frac{-b^2 + 4ac}{4a} \right)$$

### 8. Coordinates of the vertex

$$\left( \frac{-b}{2a}, \frac{-b^2 + 4ac}{4a} \right)$$

### 9. Polynomials

- Degree: largest exponent
- Leading coefficient: coefficient of the highest term
- Power functions:  $y = x^n$ , graphs of them.

## 10. Sketching Graphs

- Polynomials: ...
- Rational functions: ...

- Factorize the numerator polynomial and the denominator polynomial.
- Find the vertical asymptotes.
- Find the  $x$ -intercepts.
- Find the  $y$ -intercept.
- Find the horizontal and slant asymptotes.
- Find the excluded regions.
- Analyze the behavior near the  $x$ -intercepts and asymptotes.

## 11. Exponential Function $y = b^x$

## 12. Exponential Function $y = \log_b x$

## 13. Properties of logarithms

- $\log_b b = 1$
- $\log_b 1 = 0$
- $b^{\log_b x} = x$
- $\log_b(MN) = \log_b M + \log_b N$
- $\log_b \left( \frac{M}{N} \right) = \log_b M - \log_b N$
- $\log_b x^n = n \log_b x$
- $\log_a x = \frac{\log_b x}{\log_b a}$