

1. Simplify/evaluate the following trigonometric expressions without explicitly using the calculator. Leave the solution in **exact form**. Clearly state the identity/identities used (e.g. cos-sec reciprocal identity, Pythagorean, $\sin A \pm \sin B$, sum-to-product, double angle for sin, half angle for cos, etc.). [Total: 12 points]

(a) $\sin^2(8\pi/7) + \cos^2(8\pi/7)$ [1 point]

(b) $\cos(1050^\circ)$ [2 point]

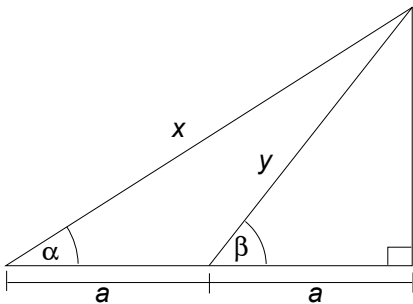
(c) $\tan(\pi/12)$ [2 point]

(d) $\cos(75^\circ) + \cos(15^\circ)$ [2 point]

(e) $\sin(37.5^\circ) \sin(7.5^\circ)$ [2 point]

(f) $\sec(15^\circ)$ (*Hint: First find $\cos(15^\circ)$*) [3 point]

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2. For the following figure, show that $y = \frac{x \cos \alpha}{2 \cos \beta}$ [Total: 3 points]



3. Prove the following identities

[Total: 12 points]

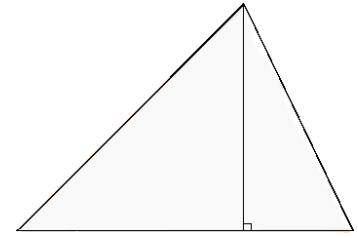
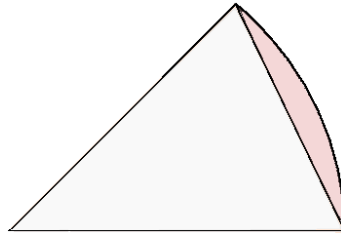
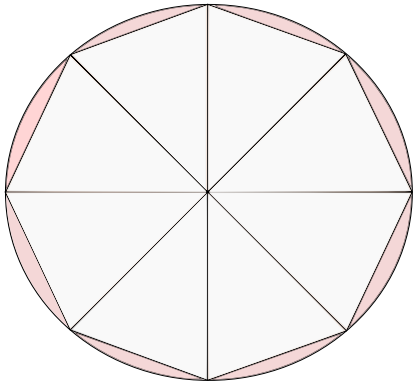
(a) $\frac{1}{1 - \cos \theta} + \frac{1}{1 + \cos \theta} = 2 + 2 \cot^2 \theta$

[6 point]

(b) $\frac{1}{2} \sin 2A = \frac{\tan A}{1 + \tan^2 A}$

[6 point]

4. A metal worker hopes to make triangular pieces of sheet metal by first taking a circular sheet of metal with an *diameter* of 12 inches, cut it in to 8 equal parts to get wedges and cut off its circular sector to finally obtain a triangular shape. Your task is to find the amount of metal wasted in this process. Answer the following questions to do this calculation. *[Total: 9 points]*



- (a) Find the radius of the circle. *[1 point]*
- (b) Find the circumference of the circular sheet of metal. (*Hint: $2\pi r$*) *[1 point]*
- (c) Find the arc length of the curved side of **one piece** of metal cut out. *[1 point]*
- (d) Find the angle (in radians) between the two straight edges of a piece of metal. *[1 point]*
- (e) Find the area of a wedge of metal cut out. *[1 point]*
- (f) Find the height of the triangular piece of metal as shown on the picture. *[1 point]*
- (g) Find the area of a triangular piece of metal finally obtained. (*Hint: $\frac{1}{2}bh$*) *[1 point]*
- (h) Find the area of metal wasted per piece. *[1 point]*
- (i) Find the area of metal wasted during the whole project. *[1 point]*

5. Consider the trigonometric function given by $y = 3 \cos\left(\frac{\pi x}{2} - \frac{\pi}{16}\right)$ *[Total: 13 points]*
- (a) Find its (fundamental) period *[1 point]*
- (b) Find its Phase shift *[1 point]*
- (c) Find its amplitude *[1 point]*
- (d) Find its vertical shift *[1 point]*
- (e) Sketch its graph for a few cycles *[3 point]*
- (f) Use the above graph to sketch the graph of $y = 3 \csc\left(\frac{\pi x}{2} - \frac{\pi}{16}\right)$ for a few cycles *[3 points]*
- (g) Use the above graph to sketch the graph of $y = 3 \csc\left(\frac{\pi|x|}{2} - \frac{\pi}{16}\right)$ for a few cycles *[3 points]*

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6. You are given that $\cos \theta = \frac{-5}{13}$ for some angle θ *[Total: 5 points]*
- (a) What quadrants can this angle θ be in? *[1 point]*
- (b) If you are also told that $\tan \theta$ is negative what quadrant can this angle θ be in? *[1 point]*
- (c) If you are also told that $\tan \theta$ is negative find the values of $\sin \theta$ and $\tan \theta$. *[3 point]*

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7. The Far-Far-Away Land has two banks. State Bank of Far-Far-Away (SBoF) and Far-Far-Away National Bank (FNB). The SBoF pays an interest rate of 7% compounded annually. The FNB on the other hand, pays 6.9% interest compounded continuously. *[Total: 6 points]*

- (a) Dronkey has invested \$ 3000 at the SBoF. How much money will be in Dronkey's account after 2 years if no subsequent transactions were made. *[3 point]*

- (b) Instead, if Dronkey invested \$ 3000 at the FNB, how long does it take to end up with the same amount that was found in part (a). *[3 point]*

8. Consider the rational function $y = \frac{2x}{(x-2)(x+2)}$ *[Total: 8 points]*

(a) Find the vertical asymptotes *[1 point]*

(b) Find the x -intercepts and y -intercepts *[1 point]*

(c) Find the horizontal asymptotes *[1 point]*

(d) Find the excluded regions *[3 point]*

(e) Use these information to sketch the graph *[2 point]*

9. Solve the following equations. ONE EXTRA POINT EACH for checking the solutions

[Total: 16 points]

(a) Solve $|3x - 9| = 6$

[3 point]

(b) Given that $f(0) = 2$, solve $3f(x^2 + 2x) - 2 = 4$

[3 point]

(c) Solve $8^3x = 16^{x^2+1}$

[3 point]

(d) Solve $\log_2(x + 5) + \log_2(2x - 9) - \log_2(x^2) = 1$

[3 point]

10. Find the domain of $f(x) = \sqrt{\frac{-(x + 2)}{(x^2 + 5x + 4)}}$

[Total: 4 points]