

**Compound Interest Formula**

Interest that is applied to the balance of an account at the end of a compounding period.

$$A = P \left( 1 + \frac{r}{n} \right)^{nt}$$

- $A = \mathbf{amount}$ : the amount in the account after interest is added
- $P = \mathbf{principal}$ : the amount in the account before interest is added
- $r = \mathbf{interest rate}$ : the interest rate also known as APR *in decimal form*
- $t = \mathbf{time}$ : the number of years
- $n = \mathbf{number of compoundings per year: number of times the interest is compounded}$ 
  - annually :  $n=1$
  - semiannually :  $n=2$
  - quarterly :  $n=4$
  - monthly :  $n=12$
  - weekly :  $n=52$
  - daily :  $n=365$
  - $\vdots$

1. In the standard notation given above, what should  $n$  be if the interest is compounded every minute.
2. If I deposit \$ 800 in a savings account at 6% interest compounded annually, and if I make no withdrawals or deposits for 4 years, how much will I have in my account at the end of 4 years?
3. Laura deposit \$ 2000 in a CD account at 3% interest compounded monthly, after 2 years, she withdraws all the money from that bank and deposits in another CD account at another bank which pays 3.5% compounded annually. How much will she have in her account 3 years after she had invested in the new bank? If she did not change the banks, how much would she have had in 5 after depositing \$ 2000 in the first bank?
4. How much should I save today in a CD account, which pays 3% compounded monthly, if I am to have \$ 3000 in 5 years?
5. Michael's parents has deposited \$ 2500 in an account which pays 2.25% interest compounded daily. If no money is deposited to or withdrawn from that account, how old will Michael be when that account has a balance of \$ 5000 ?
6. If you deposit \$ 1000 in a CD at The First United Trust Bank of Far Far Away Land, which is run by Princess Fiona and Shrek, you are guaranteed to get \$ 1500 after 5 years. All you know is they compound the interest semiannually. How much is their interest rate?

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### Continuously Compound Interest Formula

Interest that is applied to the balance of an account continuously

$$A = Pe^{rt}$$

- $A =$  **amount:** the amount in the account after interest is added
- $P =$  **principal:** the amount in the account before interest is added
- $r =$  **interest rate:** the interest rate *in decimal form*
- $e =$  **natural base:** 2.71828...
- $t =$  **time:** the number of years

### Exponential Growth and Decay Formula

A model of growth or decay that has the form

$$\mathcal{N}(t) = \mathcal{N}_0 e^{kt}$$

- $\mathcal{N}(t) =$  function notation for the size of a population at a given time
- $\mathcal{N}_0 =$  the initial population
- $e =$  **natural base:** 2.71828...
- $t =$  **time:** use the given units
- $k =$  **growth or decay constant:**
  - If  $k > 0$  growth
  - If  $k < 0$  decay

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1. Suppose you invest \$ 1000 at 4% for 5 years. Calculate (and tabulate) how much you will have as balance under different compounding plans.

Plan	$A$	$r$	$t$	$n$	$P$
Annually	\$ 1000	0.04	5	1	
Semiannually	\$ 1000	0.04	5	2	
Quarterly	\$ 1000	0.04	5	4	
Monthly	\$ 1000	0.04	5	12	
Weekly	\$ 1000	0.04	5	52	
Daily	\$ 1000	0.04	5	365	
Hourly	\$ 1000	0.04	5	$365 \times 24$	
Every minute	\$ 1000	0.04	5	$365 \times 24 \times 60$	
Every second	\$ 1000	0.04	5	$365 \times 24 \times 60 \times 60$	
Continuously	\$ 1000	0.04	5	—	

2. *Growth of bacteria can be modeled as an exponential growth.* At the start of an experiment there are 3500 bacteria present. Two hours later, the population is 5200.
- Determine the growth constant.
  - Determine the population five hours after the experiment began.
  - When will the population reach 10000?
3. *Radioactive decay can be modeled as an exponential decay.* The half-life (i.e. if you start with some amount, the time it takes to decay down to one half of the starting amount) of Iodine-131 is 8 days.
- Determine the decay constant.
  - If you start with a sample of 5 grams of Iodine-131, how much of it will remain after 6 days?
  - How long will it take until only 1 gram of Iodine-131 is left?