# MATH 2350: CALCULUS III

Spring 2011, Sections 002 & 004

Mid Term Test # 1 – Take Home Portion

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**Instructions**

- You may use any resource available (web-resources, computational resources etc.) to answer these problems.
- Some problems may involve concepts related to the course but not discussed in the class.
- Please write the solutions in your own words.
- Answer on this test book only
- Please write clearly
- Show all *necessary* work to earn full credit. You may lose points if sufficient work is not shown.
- Please return the work on **Friday, February 18th, IN CLASS**.

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Name:

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1. Consider the two planes $\mathbb{P}_1$ and $\mathbb{P}_2$ defined by $2x + z + 1 = 0$ and $4x + 3y + 2z + 1 = 0$ respectively.

   (a) How is the "angle between two planes" defined. Explain with the aid of a sketch

   (b) Find the angle between the two planes $\mathbb{P}_1$ and $\mathbb{P}_2$.

   (c) Find the coordinates of a point on the intersection of the two planes. (Sufficient to find the coordinates of only one point using any method, even trial-and-error would work.) Show that the point you found belongs to both planes.

   (d) Find the line equation of the line of intersection of the two planes $\mathbb{P}_1$ and $\mathbb{P}_2$.

2. Consider the dynamics of a particle on $\mathbb{R}^2$ using the time parameter $t$, is defined using the polar coordinates $(r, \theta)$ with $r = \mathrm{e}^{-t^2/10}$ and $\theta = t^2$. (The trace of the path of this particle gives what is known as a logarithmic spiral.)

(a) Write the polar equation (in the form $r = f(\theta)$) which describes trace of the path of the particle.

(b) Find the parametric equation of this curve in rectangular Cartesian coordinates.

(c) Find the tangent to this curve.

(d) Find the velocity components of this particle along $\boldsymbol{u}_r$ and $\boldsymbol{u}_\theta$ directions

(e) Find the acceleration components of this particle along $\boldsymbol{u}_r$ and $\boldsymbol{u}_\theta$ directions

(f) At a given time $t$, calculate how far away the particle is from the origin.

(g) Find what happens to the acceleration, velocity and distance from the origin as $t \to \infty$.

3. Evaluate the following using the techniques suggested.

(a) Given $\boldsymbol{F}(t) = \sin^2(t^2+1)\boldsymbol{i} + \cos^2(t^2+1)\boldsymbol{j} + \sqrt{2}\sin(t^2+1)\cos(t^2+1)\boldsymbol{k}$ find $\boldsymbol{F}'(t)$, the derivative of $\boldsymbol{F}(t)$ w.r.t. the parameter $t$ using the chain rule for vector valued functions.

(b) Given $\boldsymbol{F}(t) = t\left(\mathrm{e}^t + \mathrm{e}^{-t}\right)\boldsymbol{i} + t\left(\mathrm{e}^t - \mathrm{e}^{-t}\right)\boldsymbol{j}$ and $\boldsymbol{G}(t) = t\left(\mathrm{e}^t - \mathrm{e}^{-t}\right)\boldsymbol{i} - t\left(\mathrm{e}^t + \mathrm{e}^{-t}\right)\boldsymbol{j}$, let $\boldsymbol{H}(t) = \boldsymbol{F}(t) \times \boldsymbol{G}(t)$ and $h(t) = \boldsymbol{F}(t) \bullet \boldsymbol{G}(t)$. Find $\boldsymbol{H}'(t)$ and $h'(t)$ using two methods

i. First finding $\boldsymbol{H}(t)$ and $h(t)$ and differentiating w.r.t $t$

ii. Using the dot and cross product rules of differentiation

(c) Discuss the continuity and smoothness properties of the vector valued function $\boldsymbol{F}(t) = \left(\dfrac{\sin(t)}{t}\right)\boldsymbol{i} + \tan(t)\boldsymbol{j}$.

**For grading purposes only**

| 1a | 1b | 1c | 1d | 2a | 2b | 2c | 2d | 2e | 2f | 2g | 3a | 3b1 | 3b2 | 3b3 | 3b4 | 3cc | 3cs | Total | Grade |
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