MATH 2350: CALCULUS III - SPRING 2011 - REVIEW

1. Evaluate the integral $\displaystyle\int_0^1 \int_{x^2}^x \int_y^x \frac{z}{2}\, \mathrm{d}z\, \mathrm{d}y\, \mathrm{d}x$

2. Find the volume of the region bound by the plane $4x+3y+6z=12$ and the coordinate planes (i.e. $x \geq 0$, $y \geq 0$ and $z \geq 0$).

3. Find the surface area of the region of the hemisphere $z=\sqrt{4-x^2-y^2}$ contained in the contained in the cylinder $x^2+y^2=1$. *(HINT: Write the formula in rectangular coordinates and convert to polar coordinates.)*

4. Consider the sum of two integrals: $\displaystyle\int_{-4}^0 \int_{y/4}^{\sqrt{y+4}-1} xy\, \mathrm{d}x\, \mathrm{d}y + \int_0^4 \int_{y/4}^1 xy\, \mathrm{d}x\, \mathrm{d}y$. Note that both integrals have the same integrand.
   (a) Sketch the domain of integration.
   (b) Change the order of integration to $\mathrm{d}y\, \mathrm{d}x$. You will see that it is possible to write it as a single integral. *(HINT: Solve $x=\sqrt{y+4}-1$ for y)*

5. Use cylindrical coordinates to find the volume between the two paraboloids $z=4-x^2-y^2$ and $z=-4+x^2+y^2$. *(HINT: Setup the integral in rectangular coordinates and convert to cylindrical coordinates.)*

6. Given $\boldsymbol{F}(x,y,z)=(yz)\,\boldsymbol{i}+(xz)\,\boldsymbol{j}+(xy)\,\boldsymbol{k}$ and $f(x,y,z)=xyz$ find the following if they exist. Otherwise say why they do not exist.
   (a) curl $\boldsymbol{F}$ (b) div $\boldsymbol{F}$ (c) curl (div $\boldsymbol{F}$) (d) $\nabla^2 f$ (e) div (grad f)

7. Evaluate line integral $\displaystyle\oint_C x^2 y\, \mathrm{d}s$ where, C is the closed path $O \to A \to B \to O$ shown in figure 1.

8. (a) Show that $\boldsymbol{F}(x,y,z)=\left(xy^2z^2\right)\boldsymbol{i}+\left(x^2yz^2\right)\boldsymbol{j}+\left(x^2y^2z\right)\boldsymbol{k}$ is a conservative vector field.
   (b) Find the scalar potential f for this vector field.
   (c) Evaluate the line integral $\displaystyle\int_C \boldsymbol{F} \bullet \mathrm{d}\boldsymbol{R}$ over the path $O \to A \to B$ given in figure 2.

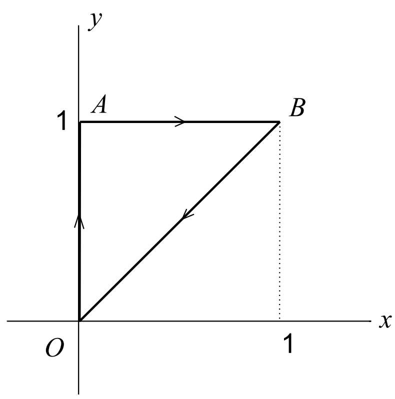

Figure 1: Figure for problem 7

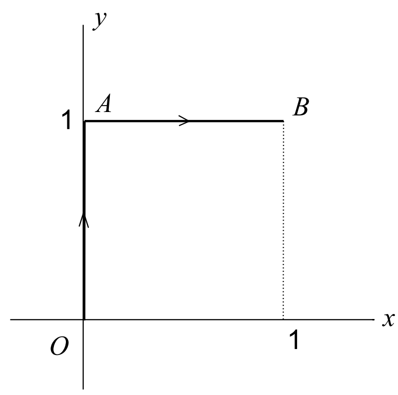

Figure 2: Figure for problem 8c

9. Use Green's theorem to evaluate $\oint_C y^2\,\mathrm{d}x + x^2\,\mathrm{d}y$, where C is the path along the circle $x^2+y^2=4$, in the counterclockwise direction.

10. Evaluate the surface integral $\iint_{\mathcal{S}} xy\,\mathrm{d}S$, where $\mathcal{S}$ is the portion of the plane $x+y+z=1$ that lies inside the cylinder $x^2+y^2=1$ and $y \geq 0$.

11. Use Stokes' theorem to evaluate $\oint_C xyz\,\mathrm{d}x + xz\,\mathrm{d}y + xy\,\mathrm{d}y$, where C is the intersection of the xy-plane and the hemisphere $z=\sqrt{1-x^2-y^2}$, traversed counterclockwise as viewed from above.

12. Use the divergence theorem to calculate $\iint_{\mathcal{S}} \boldsymbol{F} \bullet \boldsymbol{N}\,\mathrm{d}S$ where $\boldsymbol{F} = x^2\boldsymbol{i} + y^2\boldsymbol{j} + z^2\boldsymbol{k}$, ans $\mathcal{S}$ is the surface of the cube with a side 2 units and centered at the origin. (i.e. vertices at $(-2,0,0)$, $(2,0,0)$, $(0,-2,0)$, $(0,2,0)$, $(0,0,-2)$ and $(0,0,2)$)

13. (a) Find the unit tangent, unit normal and the curvature of the plane curve $\boldsymbol{R}(t) = t^2\boldsymbol{i} + t\boldsymbol{j}$.
    (b) Without any calculations, write the coordinates of the point at which this curve has maximum curvature. Give the intuitive reason for your answer.

14. The equation $\mathrm{e}^{xyz} = 2$ defines z as an implicit function of the variables x and y. Find $\dfrac{\partial^2 z}{\partial x \partial y}$.

15. Consider the surface defined by the function $z = \sin x + \sin y$.
    (a) Find the directional derivative $\mathrm{D}_{\boldsymbol{v}} z|_{(0,0,0)}$, of this function along the direction $\boldsymbol{v} = 2\boldsymbol{i} - 2\boldsymbol{j} + 1\boldsymbol{k}$, at the point $(0,0,0)$.
    (b) Identify and classify the local extrema (max, min, saddle)

16. The velocity of a particle is given by $\boldsymbol{v} = t\sin(t)\boldsymbol{i} + t\cos(t)\boldsymbol{j} + t\boldsymbol{k}$.
    (a) Find the acceleration of the particle.
    (b) If the particle started at the point $2\boldsymbol{i} + 2\boldsymbol{j} + 1\boldsymbol{k}$, at time $t=0$, find its position at time t.

17. Use the idea of the total derivative to find an approximation to $f(1.01, 0.99)$ given $f(x,y) = 2x^2y$.

18. Use the chain rule to find $\dfrac{\partial z}{\partial t}$ where $z = x^2y^3$, $x = \cos uv$, $y = \sin uv$, $u = e^t$ and $v = e^{-t}$.

19. Show that the limit of $\dfrac{x^2y}{x^2+y}$ as $(x,y) \to (0,0)$ does not exist.

20. Find the tangent plane and the normal line to the paraboloid $z = x^2+y^2$, at the point $(1,2,5)$.