MATH 2350: CALCULUS III - SPRING 2011 - MID # 3 EXTRA/MAKEUP

1. Evaluate the integral $\int_0^1 \int_{x^2}^{x} \int_y^x \frac{z}{2}\, \mathrm{d}z\, \mathrm{d}y\, \mathrm{d}x$

2. Find the volume of the region bound by the plane $4x + 3y + 6z = 12$ and the coordinate planes (i.e. $x \geq 0$, $y \geq 0$ and $z \geq 0$).

3. Find the surface area of the region of the hemisphere $z = \sqrt{4 - x^2 - y^2}$ contained in the contained in the cylinder $x^2 + y^2 = 1$. *(HINT: Write the formula in rectangular coordinates and convert to polar coordinates.)*

4. Consider the sum of two integrals: $\int_{-4}^{0} \int_{y/4}^{\sqrt{y+4}-1} xy\, \mathrm{d}x\, \mathrm{d}y + \int_0^4 \int_{y/4}^{1} xy\, \mathrm{d}x\, \mathrm{d}y$. Note that both integrals have the same integrand.
   (a) Sketch the domain of integration.
   (b) Change the order of integration to $\mathrm{d}y\, \mathrm{d}x$. You will see that it is possible to write it as a single integral. *(HINT: Solve $x = \sqrt{y+4} - 1$ for y)*

5. Use cylindrical coordinates to find the volume between the two paraboloids $z = 4 - x^2 - y^2$ and $z = -4 + x^2 + y^2$. *(HINT: Setup the integral in rectangular coordinates and convert to cylindrical coordinates.)*

6. Given $\boldsymbol{F}(x, y, z) = (yz)\, \boldsymbol{i} + (xz)\, \boldsymbol{j} + (xy)\, \boldsymbol{k}$ and $f(x, y, z) = xyz$ find the following if they exist. Otherwise say why they do not exist.
   (a) curl $\boldsymbol{F}$ (b) div $\boldsymbol{F}$ (c) curl (div $\boldsymbol{F}$) (d) $\nabla^2 f$ (e) div (grad f)

7. Evaluate line integral $\oint_C x^2 y\, \mathrm{d}s$ where, C is the closed path $O \to A \to B \to O$ shown in figure 1.

8. (a) Show that $\boldsymbol{F}(x, y, z) = \left(xy^2z^2\right) \boldsymbol{i} + \left(x^2yz^2\right) \boldsymbol{j} + \left(x^2y^2z\right) \boldsymbol{k}$ is a conservative vector field.
   (b) Find the scalar potential f for this vector field.
   (c) Evaluate the line integral $\int_C \boldsymbol{F} \bullet \mathrm{d}\boldsymbol{R}$ over the path $O \to A \to B$ given in figure 2.

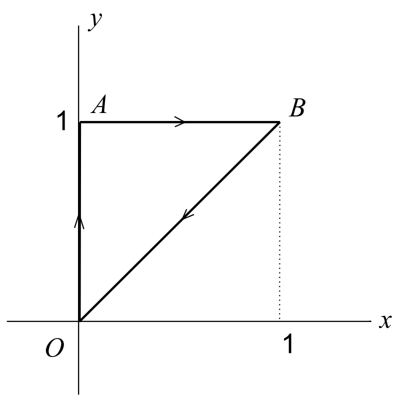

Figure 1: Figure for problem 7

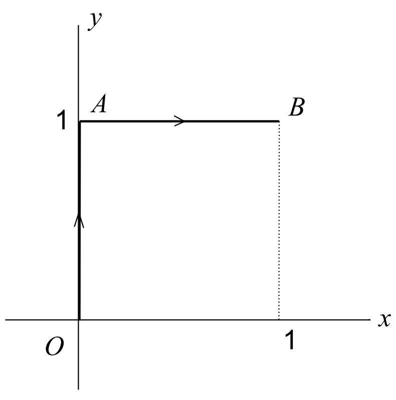

Figure 2: Figure for problem 8c