MATH 2350: CALCULUS III – Spring 2011 – Sections 002 & 004

Homework 1

Problem Set 9.3 (Page 595)

11, 12, 13, 14, 35, 36, 37, 38

Problem Set 9.4 (Page 604)

1, 2, 3, 4, 5, 6, 10, 11, 12, 15, 16, 19, 20, 23, 24, 27, 28, 29, 30, 31

Additional Problem

Resolve the vector $\boldsymbol{a}$ along the direction of the two vectors $\boldsymbol{v}$ and $\boldsymbol{w}$ as shown in the figure below.

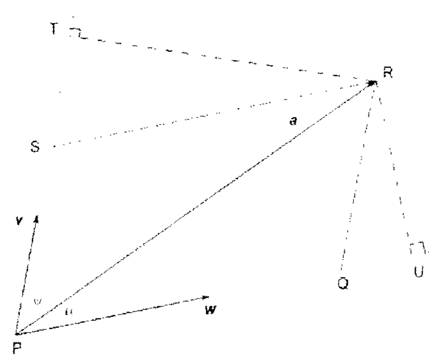
That is, find two vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ such that $\boldsymbol{a} = \boldsymbol{p} + \boldsymbol{q}$, where $\boldsymbol{p}$ is along $\boldsymbol{v}$ and $\boldsymbol{q}$ is along $\boldsymbol{w}$.

NOTE:

- The two vectors $\boldsymbol{v}$ and $\boldsymbol{w}$ are NOT orthogonal and are NOT unit vectors.
- Vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ are NOT the projections of the vector $\boldsymbol{a}$ on $\boldsymbol{v}$ and $\boldsymbol{w}$.

HINT

- Think about the geometry of the setup – refer to the figure given



$\underline{v} = 3\underline{i} - 2\underline{j} + \underline{k}$ & $\underline{w} = \underline{i} + \underline{j} - \underline{k}$

(11) $\underline{v} + \underline{w} = 4\underline{i} - \underline{j} + 0\underline{k}$

$\underline{v} - \underline{w} = 2\underline{i} - 3\underline{j} - 2\underline{k}$

$\left.\begin{array}{l} \\ \end{array}\right\} (\underline{v}+\underline{w})(\underline{v}-\underline{w}) = 8 + 3 = 11$

(12) $\underline{v} \cdot \underline{w} = 3 - 2 - 1 = 0$

$\therefore (\underline{v} \cdot w)\, \underline{w} = 0(\underline{i} + \underline{j} - \underline{k}) = 0\underline{i} + 0\underline{j} + 0\underline{k} = \underline{0}$

(13) $\|v\| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9 + 4 + 1} = \sqrt{14}$

$\|w\| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$

$(\|v\|\, w) \cdot (\|w\|\, v) = \|v\|\,\|w\|\,(w \cdot v) = (\sqrt{14})(\sqrt{3})(3 - \cancelto{0}{2 - 1}) = 0$

(14) $3\underline{v} + 2\underline{w} = 9\underline{i} - 6\underline{j} + 3\underline{k} + 2\underline{i} + 2\underline{j} - 2\underline{k} = 11\underline{i} - 8\underline{j} + \underline{k}$

$\|3v + 2w\| = \sqrt{11^2 + (-8)^2 + 1^2} = \sqrt{121 + 64 + 1} = \sqrt{186}$

$2\underline{v} + 3\underline{w} = 6\underline{i} - 4\underline{j} + 2\underline{k} + 3\underline{i} + 3\underline{j} - 3\underline{k} = 9\underline{i} - \underline{j} + 5\underline{k}$

$$\frac{2\underline{v} + 3\underline{w}}{\|3\underline{v} + 2\underline{w}\|} = \frac{9}{\sqrt{186}}\, i - \frac{1}{\sqrt{186}}\, \underline{j} + \frac{5}{\sqrt{186}}\, \underline{k}$$

CAL III – HW #01

SOLUTIONS

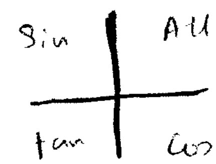
(35) $\underline{v} = \underline{i} - \underline{j} + 2\underline{k} \Rightarrow \|v\| = \sqrt{1^2 + (-1)^2 + 2^2} = \sqrt{6}$

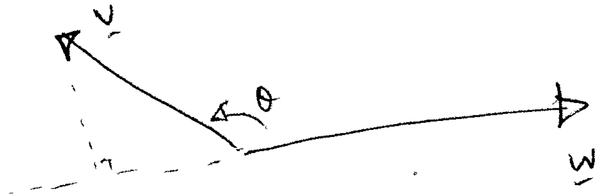
$\underline{w} = 2\underline{i} + \underline{j} - \underline{k} \Rightarrow \|w\| = \sqrt{2^2 + 1^2 + (-1)^2} = \sqrt{6}$

$\underline{v} \cdot \underline{w} = 2 - 1 - 2 = -1$

also $\underline{v} \cdot \underline{w} = \|v\|\,\|w\| \cos\theta = 6\cos\theta$

$\therefore \cos\theta = \frac{-1}{6}$





~~proj~~ $\text{proj}_{\underline{w}} \underline{v} = \frac{v \cdot w}{\|w\|} \cdot \frac{w}{\|w\|}$

$= \frac{-1}{\sqrt{6}} \cdot \frac{2\underline{i} + \underline{j} - \underline{k}}{\sqrt{6}} = -\frac{1}{6}\underline{w}$

$= \frac{-1}{3}\underline{i} - \frac{1}{6}\underline{j} + \frac{1}{6}\underline{k}$. $= \cancel{\frac{1}{6}}$

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(16)

(36) $\underline{v} = 4\underline{i} - \underline{j} + \underline{k}$, $\|v\| = \sqrt{4^2 + (-1)^2 + 1^2} = \sqrt{18}$

$\underline{w} = 2\underline{i} + 3\underline{j} - \underline{k} \Rightarrow \|w\| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{14}$

(a) $\underline{v} \cdot \underline{w} = 8 - 3 - 1 = 4$

(b) $\underline{v} \cdot \underline{w} = \|v\| \, \|w\| \cos\theta = \sqrt{18}\sqrt{14} \cos\theta$

$\sqrt{18}\sqrt{14}$
$\sqrt{2 \times 9 \times 2 \times 7}$
$= 2\sqrt{63}$

$\therefore \cos\theta = \dfrac{4}{\sqrt{18}\sqrt{14}} = \dfrac{2}{\sqrt{63}}$

(c) Set $\underline{v} \cdot (\underline{v} - s\underline{w}) = 0$ ($\because$ orthogonal)

$\underline{v} \cdot \underline{v} - s\, \underline{v} \cdot \underline{w} = 0$

$\Rightarrow \|v\|^2 - s.(4) = 0$

$18 - 4s = 0 \Rightarrow 4s = 18 \Rightarrow s = \dfrac{9}{2}$

(d) set $(s\underline{v} + \underline{w}) \cdot \underline{w} = 0$

$s\, \underline{v} \cdot \underline{w} + \underline{w} \cdot \underline{w} = 0$

$\Rightarrow s(\underline{v} \cdot \underline{w}) + \|\underline{w}\|^2 = 0$

$4s + 14 = 0$

$\Rightarrow s = -\dfrac{7}{2}$

(37) ~~Let~~ $\underline{v} = 2\underline{i} - 3\underline{j} + 6\underline{k} \Rightarrow \|v\| = \sqrt{2^2+(-3)^2+6^2} = \sqrt{4+9+36} = \sqrt{49} = 7$

$\underline{w} = 4\underline{i} + 3\underline{k} \Rightarrow \|w\| = \sqrt{4^2+3^2} = \sqrt{16+9} = \sqrt{25} = 5$

(a) $\underline{v}\cdot\underline{w} = 8 + 18 = 26$

(b) $\underline{v}\cdot\underline{w} = \|v\|\,\|w\| \cos\theta = 35\cos\theta$.

$\Rightarrow \cos\theta = \dfrac{26}{35}$

CAL III – HW #1

(c) set $(\underline{v} - s\underline{w})\cdot\underline{v} = 0$

$\|v\|^2 - s(\underline{w}\cdot\underline{v}) = 0$

$49 - 26s = 0 \Rightarrow s = \dfrac{49}{26}$

(d) set $(\underline{v} + t\underline{w})\cdot\underline{w} = 0$

$\underline{v}\cdot\underline{w} + t\|w\|^2 = 0$

$26 + 25t = 0 \Rightarrow t = \dfrac{-26}{25}$

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(38) $\operatorname{Proj}_v F = \dfrac{F\cdot V}{\|v\|}\cdot\dfrac{v}{\|v\|}$

$= \dfrac{12}{\sqrt{6}}\cdot\dfrac{(\underline{i} - \underline{j} + 2\underline{k})}{\sqrt{6}}$

$\operatorname{Proj}_v F = 2\underline{i} - 2\underline{j} + 4\underline{k}$.

$\operatorname{Comp}_v F = \dfrac{12}{\sqrt{6}} = 2\sqrt{6}$.

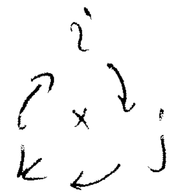
$\underline{F} = 4\underline{i} - 2\underline{j} + 3\underline{k}$

$\underline{v} = \underline{i} - \underline{j} + 2\underline{k}$

$\|\underline{v}\| = \sqrt{1+(-1)^2+2^2} = \sqrt{6}$

$\underline{F}\cdot\underline{v} = 4 + 2 + 6 = 12$

§ 9.4.



① $\underline{v} = \underline{i}$; $\underline{w} = \underline{j}$

$\therefore \underline{v} \times \underline{w} = \underline{i} \times \underline{j} = \underline{k}$

② $\underline{v} = \underline{k}$; $\underline{w} = \underline{k}$

$\therefore \underline{v} \times \underline{w} = \underline{k} \times \underline{k} = \underline{0}$

③ $\underline{v} = 3\underline{i} + 2\underline{k}$; $\underline{w} = 2\underline{i} + \underline{j}$

$(3\underline{i} + 2\underline{k}) \times (2\underline{i} + \underline{j})$

$6 \underbrace{\underline{i} \times \underline{i}}_{\underline{0}} + 3 \underbrace{\underline{i} \times \underline{j}}_{\underline{k}} + 4 \underbrace{\underline{k} \times \underline{i}}_{\underline{j}} + 2 \underbrace{\underline{k} \times \underline{j}}_{-\underline{i}}$

$-2\underline{i} + 4\underline{j} + 3\underline{k}$

④ $\underline{v} = \underline{i} - 3\underline{j}$; $\underline{w} = \underline{i} + 5\underline{k}$

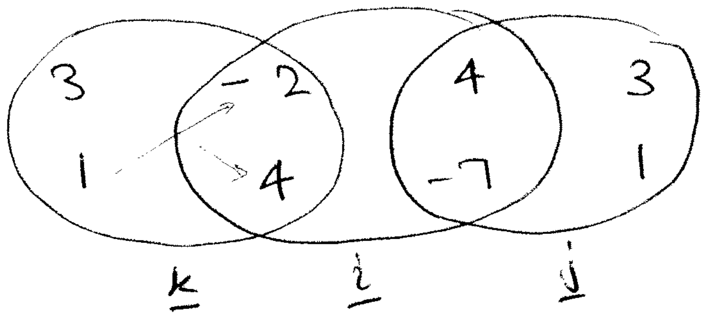
$(\underline{i} - 3\underline{j}) \times (\underline{i} + 5\underline{k})$

$\underbrace{\underline{i} \times \underline{i}}_{\underline{0}} + 5 \underbrace{\underline{i} \times \underline{k}}_{-\underline{j}} - 3 \underbrace{\underline{j} \times \underline{i}}_{-\underline{k}} - 15 \underbrace{\underline{j} \times \underline{k}}_{\underline{i}}$

$-15\underline{i} - 5\underline{j} + 3\underline{k}$

I am using the different methods (or tricks) we discussed

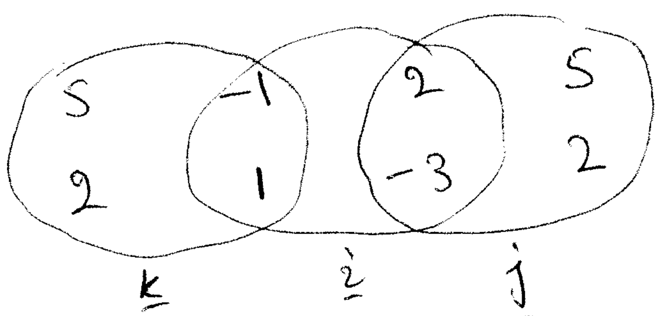
⑤ $\underline{v} = 3\underline{i} - 2\underline{j} + 4\underline{k}$; $\underline{w} = \underline{i} + 4\underline{j} - 7\underline{k}$



$$\underline{v} \times \underline{w} = (12+2)\underline{k} + (14-16)\underline{i} + (4+21)\underline{j}$$

$$\therefore \quad \underline{v} \times \underline{w} = -2\underline{i} + 25\underline{j} + 14\underline{k}$$

⑥ $\underline{v} = 5\underline{i} - \underline{j} + 2\underline{k}$; $\underline{w} = 2\underline{i} + \underline{j} - 3\underline{k}$



$$\underline{v} \times \underline{w} = (3-2)\underline{i} + (4+15)\underline{j} + (5+2)\underline{k}$$

$$= \underline{i} + 19\underline{j} + 7\underline{k}$$

* ⑧ & ⑨ were not assigned

(8) $\underline{v} = -\underline{j} + 4\underline{k}$; $\underline{w} = 5\underline{i} + 6\underline{k}$

$$\underline{v} \times \underline{w} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & -1 & 4 \\ 5 & 0 & 6 \end{vmatrix}$$

$$= \underline{i}\,(-6-0) - \underline{j}\,(0-20) + \underline{k}\,(0+5)$$

$$= -6\underline{i} + 20\underline{j} + 5\underline{k}$$

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(9) $\underline{v} = \underline{i} - 6\underline{j} + 10\underline{k}$; $\underline{w} = -\underline{i} + 5\underline{j} - 6\underline{k}$

$$\underline{v} \times \underline{w} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -6 & 10 \\ -1 & 5 & -6 \end{vmatrix}$$

$$= \underline{i}\,(36-50) - \underline{j}\,(-6+10) + \underline{k}\,(5-6)$$

$$= -14\underline{i} - 4\underline{j} - \underline{k}$$

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(10) $\underline{v} = \cos\theta\,\underline{i} + \sin\theta\,\underline{j}$; $\underline{w} = -\sin\theta\,\underline{i} + \cos\theta\,\underline{j}$

$$\underline{v} \times \underline{w} = (\cos\theta\,\underline{i} + \sin\theta\,\underline{j}) \times (-\sin\theta\,\underline{i} + \cos\theta\,\underline{j})$$

$$= -\cos\theta\sin\theta\,\underbrace{\underline{i}\times\underline{i}}_{0} + \cos^2\theta\,\underbrace{\underline{i}\times\underline{j}}_{\underline{k}} - \sin^2\theta\,\underbrace{\underline{j}\times\underline{i}}_{-\underline{k}} + \sin\theta\cos\theta\,\underbrace{\underline{j}\times\underline{j}}_{0}$$

$$= (\underbrace{\cos^2\theta + \sin^2\theta}_{=1})\,\underline{k}$$

$$= \underline{k}$$

(11) $\underline{v} = \underline{i} + \underline{k}$; $\underline{w} = \underline{i} + \underline{j}$

$$\underline{v} \times \underline{w} = (\underline{i} + \underline{k}) \times (\underline{i} + \underline{j})$$

$$= \underbrace{\underline{i} \times \underline{i}}_{\underline{0}} + \underbrace{\underline{i} \times \underline{j}}_{\underline{k}} + \underbrace{\underline{k} \times \underline{i}}_{\underline{i}} + \underbrace{\underline{k} \times \underline{j}}_{-\underline{i}}$$

$$= \underline{k}$$

$$\|v\| = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}$$

$$\|w\| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}.$$

$\|\underline{v} \times \underline{w}\| = 1$

$$\underline{v} \times \underline{w} = \|v\| \, \|w\| \sin\theta \, \underline{n}$$

$$\|\underline{v} \times \underline{w}\| = \|v\| \, \|w\| \, |\sin\theta|$$

$$\therefore |\sin\theta| = \frac{\|\underline{v} \times \underline{w}\|}{\|v\| \, \|w\|} = \frac{1}{2}$$

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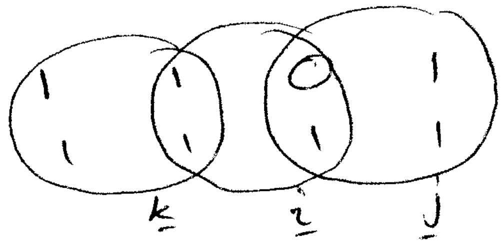
(12) $\underline{v} = \underline{i} + \underline{j} \Rightarrow \|\underline{v}\| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}$

$\underline{w} = \underline{i} + \underline{j} + \underline{k} \Rightarrow \|\underline{w}\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}.$

$\underline{v} \times \underline{w} \Rightarrow$

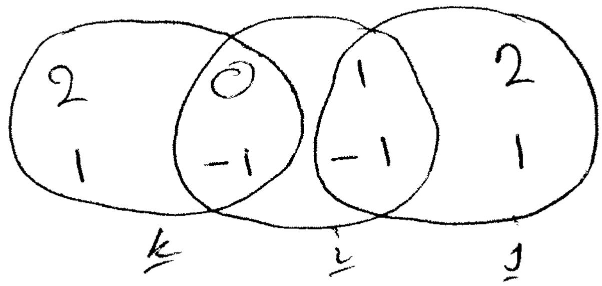


$$\therefore \underline{v} \times \underline{w} = \underline{i} - \underline{j} \Rightarrow \|\underline{v} \times \underline{w}\| = \sqrt{1^2 + (-1)^2 + 0^2} = \sqrt{2}$$

$$\therefore |\sin\theta| = \frac{\|\underline{v} \times \underline{w}\|}{\|v\| \, \|w\|} = \frac{\sqrt{2}}{\sqrt{2}\sqrt{3}} = \frac{1}{\sqrt{3}}$$

15) $\underline{v} = 2\underline{i} + \underline{k}$; $\underline{w} = \underline{i} - \underline{j} - \underline{k}$

$\underline{v} \times \underline{w} \Rightarrow$



$$\underline{v} \times \underline{w} = \underline{i} + 3\underline{j} - 2\underline{k}$$

$$\|\underline{w} \times \underline{w}\| = \sqrt{1 + 9 + 4} = \sqrt{14}$$

$$\therefore \frac{\underline{v} \times \underline{w}}{\|\underline{v} \times \underline{w}\|} = \frac{1}{\sqrt{14}}\underline{i} + \frac{3}{\sqrt{14}}\underline{j} - \frac{2}{\sqrt{14}}\underline{k}$$

Check:

$$\underline{v} \cdot \left(\frac{\underline{v} \times \underline{w}}{\|\underline{v} \times \underline{w}\|}\right) = \frac{2}{14} + 0 - \frac{2}{14} = 0$$

$$\underline{w} \cdot \left(\frac{\underline{v} \times \underline{w}}{\|\underline{v} \times \underline{w}\|}\right) = \frac{1}{\sqrt{14}} - \frac{3}{\sqrt{14}} + \frac{2}{\sqrt{14}} = 0$$

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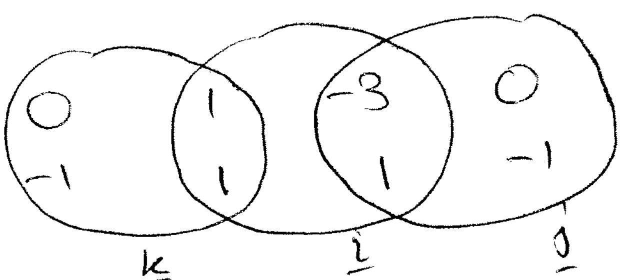
16) $\underline{v} = \underline{j} - 3\underline{k}$; $\underline{w} = -\underline{i} + \underline{j} + \underline{k}$



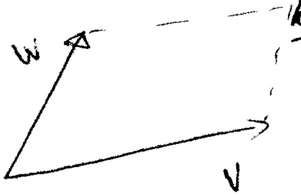
$\Rightarrow \underline{v} \times \underline{w} = 4\underline{i} + 3\underline{j} + \underline{k}$

$$\|\underline{v} \times \underline{w}\| = \sqrt{16 + 9 + 1} = \sqrt{26}$$

$$\frac{\underline{v} \times \underline{w}}{\|\underline{v} \times \underline{w}\|} = \frac{4}{\sqrt{26}}\underline{i} + \frac{3}{\sqrt{26}}\underline{j} + \frac{1}{\sqrt{26}}\underline{k}$$

(19) $\underline{v} = 3\underline{i} + 4\underline{j}$; $\underline{w} = \underline{i} + \underline{j} - \underline{k}$

$$\begin{array}{cccc} 3 & 4 & 0 & 3 \\ 1 & 1 & -1 & 1 \\ \underline{k} & \underline{i} & \underline{j} & \end{array} \Rightarrow \underline{v} \times \underline{w} = -4\underline{i} + 3\underline{j} - \underline{k}$$



$$\text{Area} = \|\underline{v} \times \underline{w}\| = \sqrt{16+9+1} = \sqrt{26}$$

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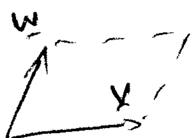
(20) $\underline{v} = 2\underline{i} - \underline{j} + 2\underline{k}$; $\underline{w} = 4\underline{i} - 3\underline{j}$

$$\begin{array}{cccc} 2 & -1 & 2 & 2 \\ 4 & -3 & 0 & 4 \\ \underline{k} & \underline{i} & \underline{j} & \end{array} \Rightarrow \underline{v} \times \underline{w} = 6\underline{i} + 8\underline{j} - 2\underline{k}$$

$$\|\underline{v} \times \underline{w}\| = \sqrt{36+64+4}$$



$$\text{Area} = \sqrt{104}$$

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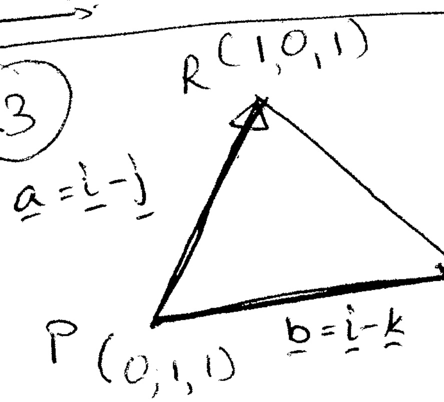
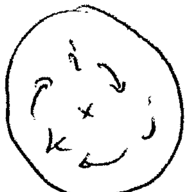
(23)





$$\underline{a} \times \underline{b} = (\underline{i} - \underline{j}) \times (\underline{i} - \underline{k})$$

$$= \underbrace{\underline{i} \times \underline{i}}_{0} - \underbrace{\underline{i} \times \underline{k}}_{-\underline{j}} - \underbrace{\underline{j} \times \underline{i}}_{-\underline{k}} + \underbrace{\underline{j} \times \underline{k}}_{\underline{i}}$$

$$= \underline{i} + \underline{j} + \underline{k}$$

$$\|\underline{a} \times \underline{b}\| = \sqrt{1+1+1} = \sqrt{3}$$

$$\text{Area} = \frac{1}{2}\|\underline{a} \times \underline{b}\| = \frac{\sqrt{3}}{2}$$

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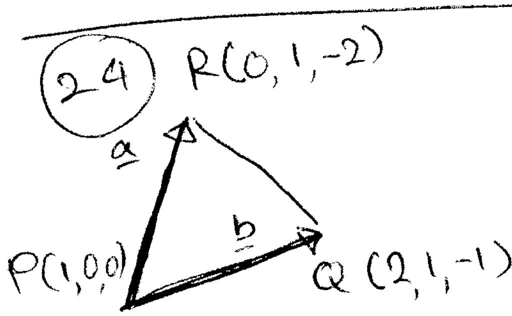
(24)



$\underline{a} = -\underline{i} + \underline{j} - 2\underline{k}$; $\underline{b} = \underline{i} + \underline{j} - \underline{k}$

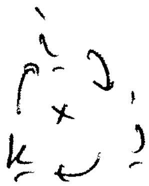
$$\underline{a} \times \underline{b} = (-1+2)\underline{i} + (-2-1)\underline{j} + (-1-1)\underline{k}$$

$$= \underline{i} - 3\underline{j} - 2\underline{k}$$

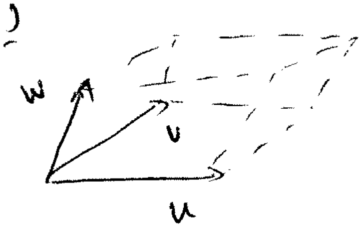
$$\therefore \text{Area} = \frac{1}{2}\sqrt{1+9+4} = \frac{\sqrt{14}}{2}$$

(30) $\underline{u} = \underline{i} + \underline{j}$; $\underline{v} = \underline{j} + 2\underline{k}$, $\underline{w} = 3\underline{k}$

~~(u×v)~~ or $\underline{u} \times \underline{v} = (\underline{i} + \underline{j}) \times (\underline{j} + 2\underline{k})$



$$= \underline{i} \times \underline{j} + 2\,\underline{i} \times \underline{k} + \underbrace{\underline{j} \times \underline{j}}_{0} + 2\,\underline{j} \times \underline{k}$$



$$= \underline{k} - 2\underline{j} + 2\underline{i}$$

$$= 2\underline{i} - 2\underline{j} + \underline{k}$$

$(\underline{u} \times \underline{v}) \cdot \underline{w} = 3$ = Area of the parallelopiped

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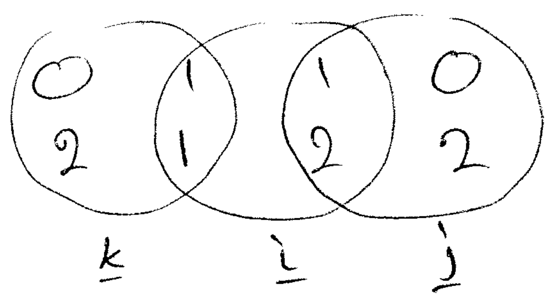
(31) $\underline{u} = \underline{j} + \underline{k}$; $\underline{v} = 2\underline{i} + \underline{j} + 2\underline{k}$; $\underline{w} = 5\underline{i}$

$\underline{u} \times \underline{v} \Rightarrow$



$$\underline{u} \times \underline{v} = (2-1)\underline{i} + 2\underline{j} - 2\underline{k}$$

$$\underline{u} \times \underline{v} = \underline{i} + 2\underline{j} - 2\underline{k}$$

$(\underline{u} \times \underline{v}) \cdot \underline{w} = 5$ = Area of the parallelopiped.

27 (a) $\underline{u} \times (\underbrace{\underline{v} \cdot \underline{w}}_{\text{Scalar}})$ Does not exist. (We cannot take a "cross product" between a vector & a scalar)

(b) $\underline{u} \cdot (\underbrace{\underline{v} \times \underline{w}}_{\text{vector}})$ Exists.

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28 (a) $\underline{u} \times (\underbrace{\underline{v} \times \underline{w}}_{\text{vector}})$ Exists ; infact,

$$\underline{u} \times (\underline{v} \times \underline{w}) = (\underline{w} \cdot \underline{u})\, \underline{v} - (\underline{v} \cdot \underline{u})\, \underline{w}$$

(b) $\underline{u} \cdot (\underbrace{\underline{v} \cdot \underline{w}}_{\text{Scalar}})$ Does not exists.

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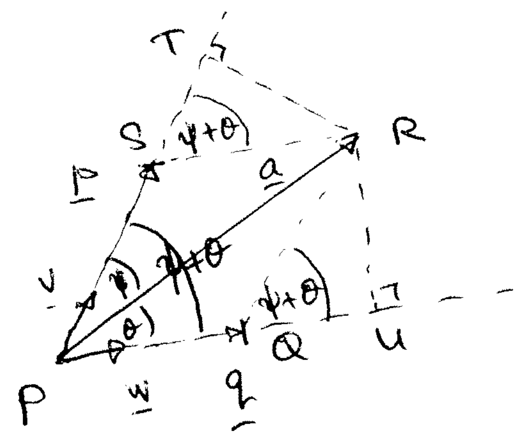
29 (a) $(\underbrace{\underline{u} \times \underline{v}}_{\text{Vector}}) \cdot (\underbrace{\underline{u} \times \underline{w}}_{\text{vector}})$ Exists

(b) $(\underbrace{\underline{u} \times \underline{v}}_{\text{Vector}}) \times (\underbrace{\underline{u} \times \underline{w}}_{\text{Vector}})$ Exists.

Infact; $(\underline{u} \times \underline{v}) \times (\underline{u} \times \underline{w}) = [\underline{w} \cdot (\underline{u} \times \underline{v})]\, \underline{u} - [\underline{u} \cdot (\underline{u} \times \underline{v})]\, \underline{w}$

Additional Problem:



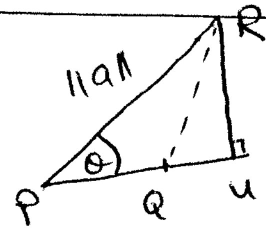
Length of diagonal $PR = \|\underline{a}\|$

PQRS is a parallelogram with $PS \parallel QR$ and $PQ \parallel SR$. Also, length of PS = length QR and, length of PQ = length of SR

Length of QR = length of PS = $\|\underline{p}\|$

Length of RS = Length of PQ = $\|\underline{q}\|$

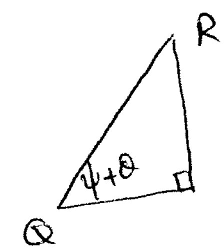
First note that



Length of RU = $\|\underline{a}\| \sin\theta$

$$= \frac{\|\underline{a}\| \|\underline{w}\| \sin\theta}{\|\underline{w}\|}$$

$$= \frac{\|a \times w\|}{\|w\|}.$$



On the otherhand, length of RU = $QR \sin(\psi+\theta)$

$$= \|\underline{p}\| \sin(\psi+\theta)$$

Note that, $\sin(\psi+\theta) = \dfrac{\|v\|\|w\| \sin(\psi+\theta)}{\|v\| \|w\|}$

$$= \frac{\|v \times w\|}{\|v\| \|w\|}$$

$\therefore$ Length of RU $= \|\underline{p}\| \cdot \dfrac{\|\underline{v} \times \underline{w}\|}{\|\underline{v}\| \, \|\underline{w}\|}$

So,

$$\frac{\|\underline{a} \times \underline{w}\|}{\|\underline{w}\|} = \|\underline{p}\| \; \frac{\|\underline{v} \times \underline{w}\|}{\|\underline{v}\| \, \|\underline{w}\|}$$

Hence,

$$\|\underline{p}\| = \frac{\|\underline{a} \times \underline{w}\|}{\|\underline{v} \times \underline{w}\|} \cdot \|v\|$$

~~So~~ Since, $\underline{p}$ is along ~~#~~ $\underline{v}$, multiply $\|\underline{p}\|$ by $\underline{v}/\|\underline{v}\|$; the unit vector along $\underline{v}$

$$\underline{p} = \frac{\|\underline{a} \times \underline{w}\|}{\|\underline{v} \times \underline{w}\|} \cancel{\|\underline{v}\|} \cdot \frac{\underline{v}}{\cancel{\|\underline{v}\|}}$$

$$\underline{p} = \frac{\|\underline{a} \times \underline{w}\|}{\|\underline{v} \times \underline{w}\|} \, \underline{v}$$

~~q~~ Similarly, $\underline{q} = \dfrac{\|\underline{a} \times \underline{v}\|}{\|\underline{v} \times \underline{w}\|} \, \underline{w}$