MATH 2350: CALCULUS III – Spring 2011 – Sections 002 & 004

Homework 2

Problem Set 9.5 (Page 614)

3, 4, 9, 10, 11, 12, 15, 16, 17 – 38

Problem Set 9.3 (Page 595)

31, 32, 33, 34

Problem Set 9.6 (Page 621)

8, 9, 14, 15, 18, 19, 20, 21, 22 23, 24, 25, 29, 30 32, 33, 34, 54, 37 (Solution to this problem is given but I need you to work this problem. Show all necessary work.)

Instructions:

- Odd numbered problems are "assigned" as examples for you to try and compare with the solutions provided at the back of the book.
- It is sufficient to provide solutions only to the even numbered problems, unless otherwise stated. I will grade a few randomly chosen problems out of them.

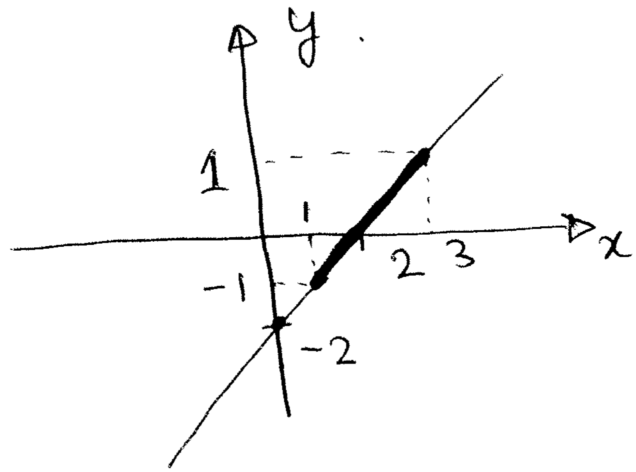
③ ~~$y = x+1$~~ , $x = t+1$, $y = t-1$; $0 \le t \le 2$

So, $t = x-1$; $t = y+1$.

$\Rightarrow x-1 = y+1$

$y = x-2$

t	x	y
0	1	-1
1	2	0
2	3	1

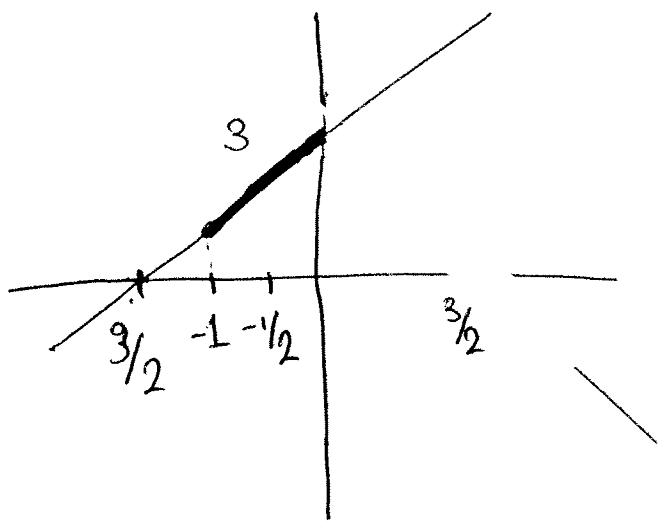


④ $x = -t$; $y = 3-2t$; $0 \le t \le 1$

So, $2x = -2t$

$y = 3+2x$

t	x	y
0	0	3
1/2	-1/2	2
1	-1	1



§9.5

⑨ $x = 3\cos\theta$; $y = 3\sin\theta$; $0 \le \theta \le 2\pi$

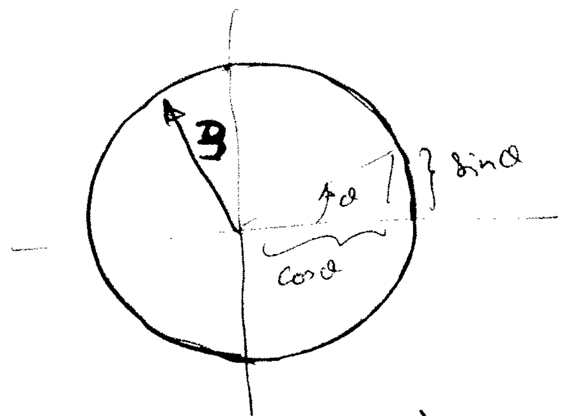
$$\boxed{\text{Hint: } \cos^2\theta + \sin^2\theta = 1}$$

$\frac{x}{3} = \cos\theta$; $\frac{y}{3} = \sin\theta$.

$$\cos^2\theta + \sin^2\theta = 1$$

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$$

$$\Rightarrow x^2 + y^2 = 9$$



(whole circle)

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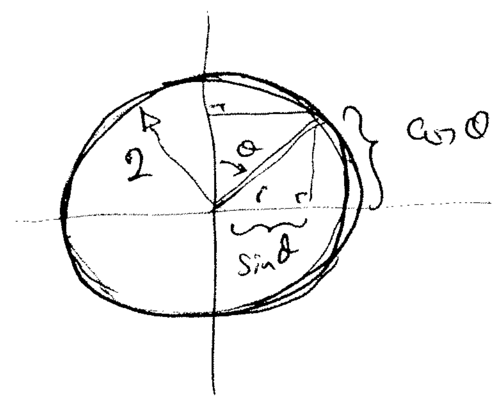
⑩ $x = 2\sin\theta$; ~~$y = 2\cos\theta$~~

$y = 2\cos\theta$;

$0 \le \theta \le 2\pi$

Similar to above

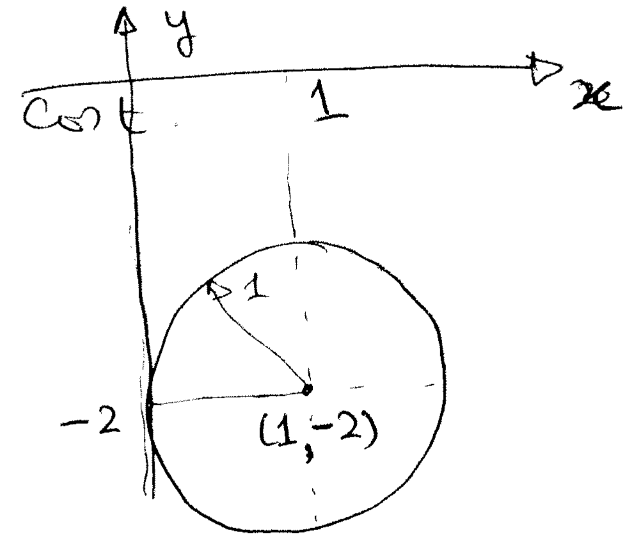


(11) $x = 1 + \sin t$; $y = -2 + \cos t$; $0 \le t \le 2\pi$

$x - 1 = \sin t$; $y + 2 = \cos t$

$\sin^2 t + \cos^2 t = 1$.

$(x-1)^2 + (y+2)^2 = 1$



(12) $x = 1 + \sin^2 t$; $y = -2 + \cos t$; $0 \le t \le \pi$

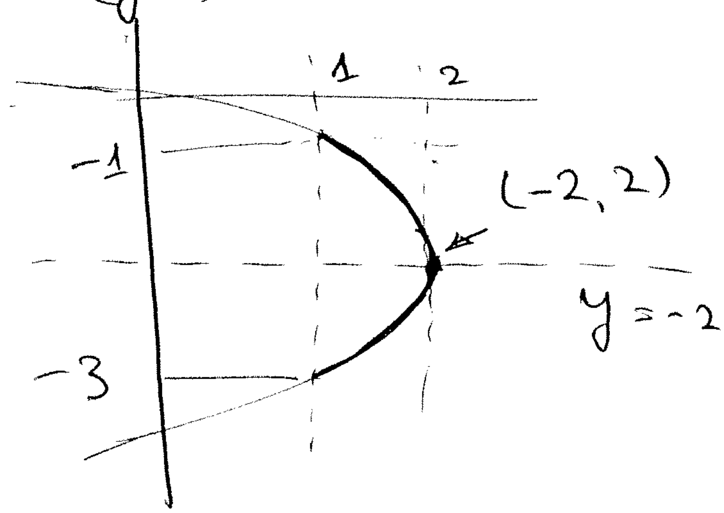
$x - 1 = \sin^2 t$, $y + 2 = \cos^2 t$.

$\sin^2 t + \cos^2 t = 1$

$(x-1) + (y+2)^2 = 1$.

So, $x = 2 - (y+2)^2$

t	x	y
0	1	-1
$\pi/2$	2	-2
π	1	-3



(15) $x = t^3$; $y = 3\ln t$ $\quad t > 0$

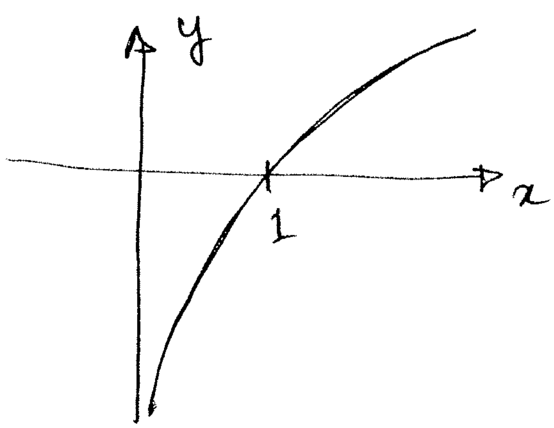
$\frac{y}{3} = \ln t \Rightarrow e^{y/3} = e^{\ln t} = t$.

So, $x = (\underbrace{e^{y/3}}_{t})^3 = e^{3y/3} = e^y$

$x = e^y \Rightarrow y = \ln x$; $x > 0$.

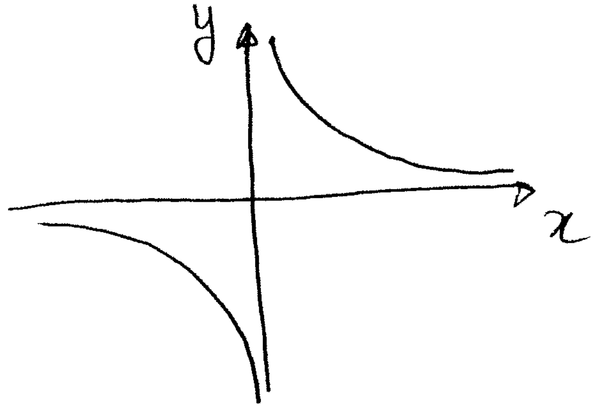
Or, $\sqrt[3]{x} = t \Rightarrow y = 3\ln\sqrt[3]{x} = 3 \cdot \frac{1}{3}\ln x$

$y = \ln x$. , $x > 0$.



(16) $x = e^t$; $y = e^{-t} = \frac{1}{e^t} = \frac{1}{x}$

$y = \frac{1}{x}$



(17) $P(1,-1,-2)$; $\underline{v} = 3\underline{i} - 2\underline{j} + 5k$

Parametric eq. $x = 1+3t$

$y = -1-2t$

$z = -2+5t$

Symmetric form: $\dfrac{x-1}{3} = \dfrac{y+1}{-2} = \dfrac{z+2}{5}$

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(18) $P(1,0,-1)$; $\underline{v} = 3\underline{i} + 4\underline{j} + 0\underline{k}$

Parametric Eq. $x = 1+3t$

$y = 0+4t$

$z = -1+0t = -1$

Cannot write the symmetric form.

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(19) $P_1(1,-1,2)$, $P_2(2,1,3)$

$\underline{v} = (2-1)\underline{i} + (1-(-1))\underline{j} + (3-2)\underline{k}$

$\underline{v} = 1\underline{i} + 2\underline{j} + 1\underline{k}$

Parametric eq. $x = 1+t$; $y = -1+2t$; $z = 2+t$

Symmetric form: $\dfrac{x-1}{1} = \dfrac{y+1}{2} = \dfrac{z-2}{1}$

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or: $x = 2+t$; $y = 1+2t$; $z = 3+t$

Symmetric form $\dfrac{x-2}{1} = \dfrac{y-1}{2} = \dfrac{z-3}{1}$

(20) $P_1 (2,2,3)$; $P_2 (1,3,-1)$

$\underline{v} = (1-2)\underline{i} + (3-2)\underline{j} + (-1-3)\underline{k}$

$\underline{v} = -1\underline{i} + 1\underline{j} - 4\underline{k}$

Param: $x = 2 - t$; $y = 2 + t$; $z = 3 - 4t$

Sym: $\dfrac{x-2}{-1} = \dfrac{y-2}{1} = \dfrac{z-3}{-4}$

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or

Param: $x =$ [redacted] $- t$; $y = 3 + t$; $z = -1 - 4t$

Sym. $\dfrac{x-1}{-1} = \dfrac{2y-3}{1} = \dfrac{z+1}{-4}$

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(21) $P(1,-3,6)$ Parallel to $\dfrac{x-5}{1} = \dfrac{y+2}{-3} = \dfrac{z}{-5}$

$\underline{v} = 1\underline{i} - 3\underline{j} - 5\underline{k}$

Param: $x = 1 + t$; $y = -3 - 3t$; $z = 6 - 5t$

Sym. $\dfrac{x-1}{1} = \dfrac{y+3}{-3} = \dfrac{z-6}{-5}$

(22) $P(1,-1,2)$; Parallel to $\frac{x+3}{4} = \frac{y-2}{5} = \frac{z+5}{1}$

$\underline{v} = 4\underline{i} + 5\underline{j} + 1\underline{k}$

Param: $x = 1+4t$; $y = -1+5t$; $z = 2+t$.

Sym. $\frac{x-1}{4} = \frac{y+1}{5} = \frac{z-2}{1}$.

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(23) $P(0,4,-3)$ Parallel to $\frac{x-1}{22} = \frac{y+2}{-6} = \frac{z-1}{10}$

$\underline{v} = 22\underline{i} - 6\underline{j} + 10\underline{k}$

In fact we can simplify this to

$\underline{v} = 11\underline{i} - 3\underline{j} + 5\underline{k}$.

So, Param: $x = 11t$; $y = 4-3t$; $z = -3+5t$

Sym. $\frac{x}{11} = \frac{y-4}{-3} = \frac{z+3}{5}$

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(24) $P(1,0,-4)$ Parallel to $\left.\begin{aligned} x &= -2+3t \\ y &= 4+t \\ z &= 2+2t \end{aligned}\right\} \underline{v} = 3\underline{i} + 1\underline{j} + 2\underline{k}$

Param: $x = 1+3t$; $y = t$; $z = -4+2t$

Sym: $\frac{x-1}{3} = \frac{y}{1} = \frac{z+4}{2}$

(25) $P(-1, 1, 6)$ Perpendicular to $3x + y - 2z = 5$

~~A~~ Note that this line will be parallel to the normal to the given plane (Because the normal is perpendicular to the plane).

So, $\underline{v} = \underline{N} = 3\underline{i} + 1\underline{j} - 2\underline{k}$

So, parametric Eq. $\Rightarrow x = -1 + 3t$,

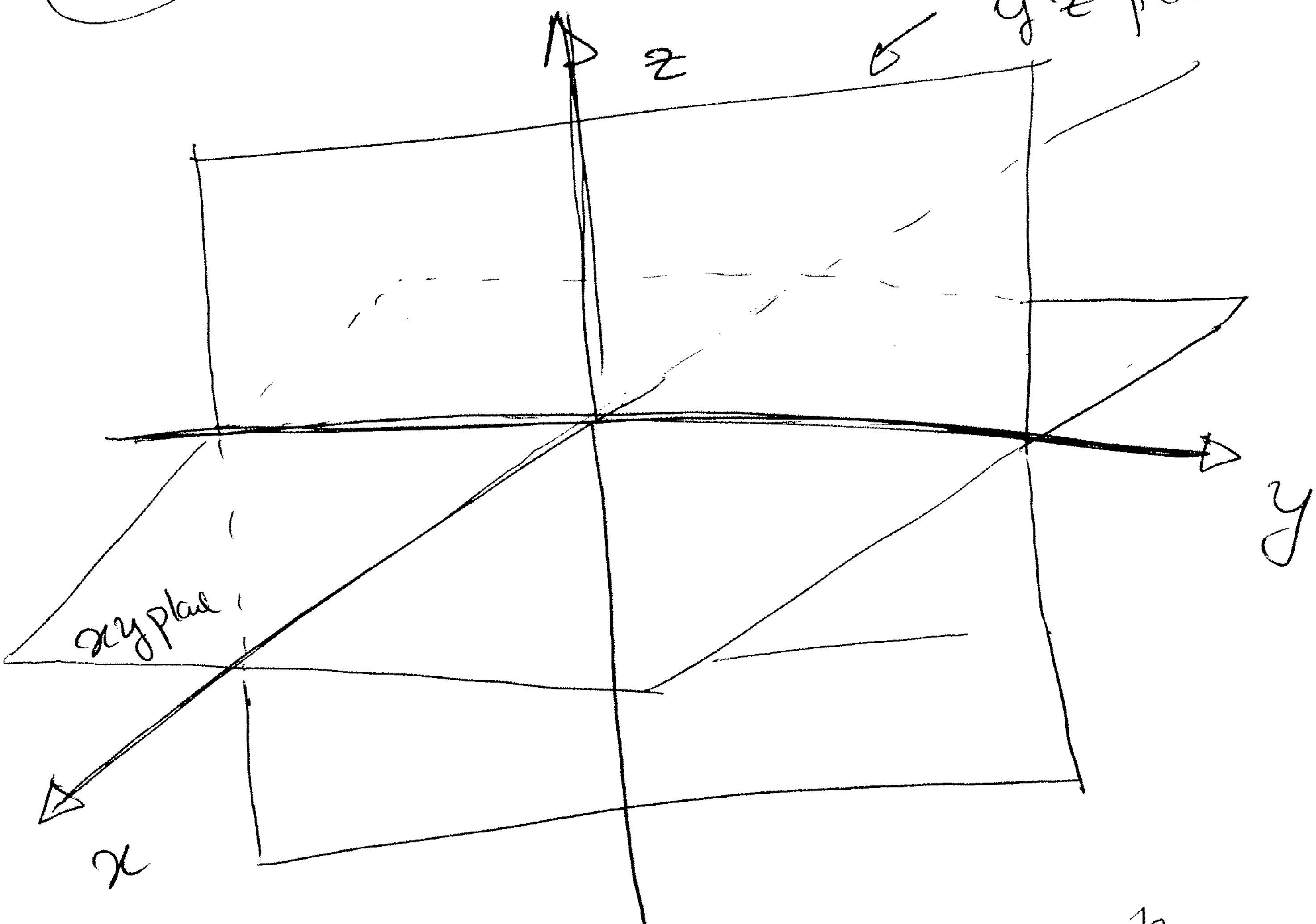
$y = 1 + t$

$z = 6 - 2t$.

Symmetric form:

$$\frac{x+1}{3} = \frac{y-1}{1} = \frac{z-6}{-2}$$

26 $P(3,-1,0)$



so, the common line is the y-axis,

so, $\underline{v} = 0\underline{i} + 1\underline{j} + 0\underline{k}$

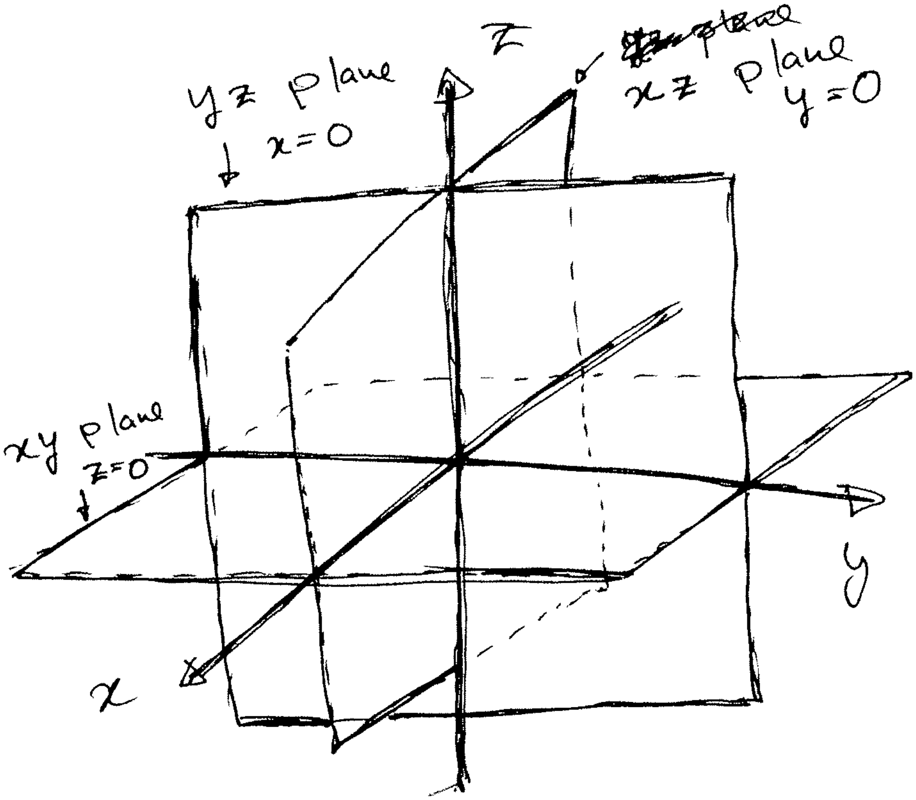
Parametric Equation:

$x = 3$; $y = -1 + t$; $z = 0$

No symmetric form.

27

$$\frac{x-4}{4} = \frac{y+3}{3} = \frac{z+2}{1}$$



Think about the parametric equation:

$x = 4t + 4$

$y = 3t - 3$

$z = \cancel{2}\, t - 2$

xy plane $\Rightarrow$ $z = t - 2 = 0 \Rightarrow t = 2$,

$$\left.\begin{aligned} &\text{So, } x = (4)(2) + 4 = 12 \\ &y = (3)(2) - 3 = 3 \end{aligned}\right\} (12, 3, 0)$$

yz plane $\Rightarrow$ $x = 0 = 4t + 4 \Rightarrow t = -1$.

$$\left.\begin{aligned} &\text{So, } y = (3)(-1)\cancel{4} - 3 = -6 \\ &z = (-1) - 2 = -3 \end{aligned}\right\} (0, -6, -3)$$

xz plane $\Rightarrow$ $y = 0 = 3t - 3 \Rightarrow t = 1$.

$$\left.\begin{aligned} &\text{So, } x = (4)(1) + 4 = 8 \\ &z = (1) - 2 = -1 \end{aligned}\right\} (8, 0, -1)$$

Let's list 27 - 30 as follows:

#	Symmetric Eq.	Parametric Eq.	Plane	Known Value	t-Value	x	y	z
27	$\frac{x-4}{4}=\frac{y+3}{3}=\frac{z+2}{1}$	$x=4t+4$, $y=3t-3$, $z=t+2$	xy	$z=0$	$t-2=0$, $t=2$	$(4)(2)+4=12$	$(3)(2)-3=3$	0
			yz	$x=0$	$4t+4=0$, $t=-1$	0	$(3)(-1)-3=-6$	$-1-2=-3$
			zx	$y=0$	$3t-3=0$, $t=1$	$(4)(1)+4=8$	0	$1-2=-1$
28	$\frac{x+1}{1}=\frac{y+2}{2}=\frac{z-6}{3}$	$x=t-1$, $y=2t-2$, $z=3t+6$	xy	$z=0$	$3t+6=0$, $t=-2$	$-2-1=-3$	$(2)(-2)-2=-6$	0
			yz	$x=0$	$t-1=0 \Rightarrow t=1$	0	$(2)(1)-2=0$	$(3)(1)+6=9$
			zx	$y=0$	$2t-2=0 \Rightarrow t=1$	$1-1=0$	0	$(3)(1)+6=9$
29	Not given	$x=6-2t$, $y=1+t$, $z=3t$	xy	$z=0$	$3t=0 \Rightarrow t=0$	6	1	0
			yz	$x=0$	$6-2t=0$, $t=3$	0	4	9
			zx	$y=0$	$1+t=0 \Rightarrow t=-1$	8	0	-3
30	Not given	$x=6+3t$, $y=2-t$, $z=2t$	xy	$z=0$	$2t=0 \Rightarrow t=0$	6	2	0
			yz	$x=0$	$6+3t=0 \Rightarrow t=-2$	0	4	-4
			zx	$y=0$	$2-t=0 \Rightarrow t=2$	12	0	4

A small note:

If two lines are coinsident, then, <u>any point</u> which satisfies the first line will satisfy the second line as well.

So, we can ~~set~~ pick a point from the first line and check if it is on the ~~sec~~ second line.

For an equation of a line given in the symmetric form, you can plug-in to the equation.

If the equation is in the parametric form, use one coordinate to find the value of the parameter and see if you get the same point.

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31 - 36 First Let's list down the details

31	$\frac{x-4}{2} = \frac{y-6}{-3} = \frac{z+2}{5}$ $\frac{x}{4} = \frac{y+2}{-6} = \frac{z-3}{10}$	$\underline{v}_1 = 2\underline{i} - 3\underline{j} + 5\underline{k}$ $\underline{v}_2 = 4\underline{i} - 6\underline{j} + 10\underline{k}$	$\underline{v}_2 = 2\underline{v}_1$	Can be Parallel or Coinsident
32	$x = 4-2t;\ y = 6t;\ z = 7-4t$ $x = 5+t;\ y = 1-3t;\ z = -3+2t$	$\underline{v}_1 = -2\underline{i} + 6\underline{j} - 4\underline{k}$ $\underline{v}_2 = 1\underline{i} - 3\underline{j} + 2\underline{k}$	$\underline{v}_1 = -2\underline{v}_2$	Can be parallel or Coinsident
33	$x = 3+3t;\ y = 1-4t;\ z = -4-7t$ $x = 2+3t;\ y = 5-4t;\ z = 3-7t$	$\underline{v}_1 = 3\underline{i} - 4\underline{j} - 7\underline{k}$ $\underline{v}_2 = 3\underline{i} - 4\underline{j} - 7\underline{k}$	$\underline{v}_1 = \underline{v}_2$	Can be Parallel or Coinsident
34	$x = 2-4t;\ y = 1+t;\ z = \frac{1}{2} + 5t$ $x = 3-t;\ y = -2-t;\ z = 4-2t$	$\underline{v}_1 = -4\underline{i} + 1\underline{j} + 5\underline{k}$ $\underline{v}_2 = 3\underline{i} - 1\underline{j} - 2\underline{k}$		Can be skew or intersect
35	$\frac{x-3}{2} = \frac{y-1}{-1} = \frac{z-4}{1}$ $\frac{x+2}{3} = \frac{y-3}{-1} = \frac{z-2}{1}$	$\underline{v}_1 = 2\underline{i} - 1\underline{j} + 1\underline{k}$ $\underline{v}_2 = 3\underline{i} - 1\underline{j} + 1\underline{k}$		Can be skew or intersect
36	$\frac{x+1}{2} = \frac{y-3}{-1} = \frac{z-2}{1}$ $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z-3}{-4}$	$\underline{v}_1 = 2\underline{i} - 1\underline{j} + 1\underline{k}$ $\underline{v}_2 = 2\underline{i} + 3\underline{j} - 4\underline{k}$		Can be skew or intersect

(31) Clearly $(4, 6, -2)$ is on the first line. Substitute it to the equation of the second line:

$$x \Rightarrow \frac{4}{4} = 1 \quad ; \quad y \Rightarrow \frac{6+2}{-6} = \frac{8}{-6} = -\frac{4}{3} \quad ; \quad z = \frac{-2-3}{10} = -\frac{1}{2}$$

$$1 \neq -\frac{4}{3} \neq -\frac{1}{2}.$$

So, it is not on the second line.

$\therefore$ The two lines are only parallel (NOT coinsident)

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(32) Clearly $(4, 0, 7)$ is on the first line.

From the second line:

$$\left.\begin{array}{l} x \Rightarrow \; 5 + t = 4 \Rightarrow t = 1. \\ \text{this gives, } y = 1 - 3t \Rightarrow 1 - 3 = -2 \\ z = -3 + 2t \Rightarrow -3 + 2 = -1 \end{array}\right\} \text{ you get } (4, -2, -1). \text{ DIFFEREN!}$$

So, the point $(4, 0, 7)$ is not on the second line.

So parallel Not Coinsident

(33) $(3, 1, -4)$ is on the first line.

$x \Rightarrow \cancel{3}\ 2+3t = 3 \Rightarrow 3t = 1 \Rightarrow t = \frac{1}{3}$

~~So~~ This gives $y = 5 - \frac{4}{3} = \frac{15-4}{3} = \frac{11}{3}$

$z = 3 - \frac{7}{3} = \frac{9-7}{3} = \frac{2}{3}$

$(3, \frac{11}{3}, \frac{2}{3})$

So, $(3,1,-4)$ is not on the second line

Only Parallel.

To find the points of intersection, rename the parameter of one line and solve!

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(34) $x \Rightarrow 2 - 4t = 3s$ — (1)

$y \Rightarrow 1 + t = -2 - s$ — (2)

$z \Rightarrow \frac{1}{2} + 5t = 4 - 2s$ — (3)

from (2) $\Rightarrow t = -3 - s$ — (4)

(1) $\Rightarrow 2 - 4(-3-s) = 3s \Rightarrow 2 + 12 - 4s = 3s$.

$\Rightarrow 7s = 14$

$\Rightarrow s = 2$

So, (4) $\Rightarrow t = -3 - 2 = 5$

Since we did not use (3) : $\frac{1}{2} + (5)(5) = 25\frac{1}{2} \neq \underbrace{4 - 2(2)}_{=0}$

So, they donot intersect.

(35) ~~Line 1~~: parametric Equations:

Line 1:	$x = 3 + 2t$	$y = 1 - t$	$z = 4 + t$
Line 2:	$x = -2 + 3s$	$y = 3 - s$	$z = 2 + s$

$3 + 2t = -2 + 3s$ —— (1)

$1 - t = 3 - s$ —— (2)

$4 + t = 2 + s$ —— (3)

(3) $\Rightarrow$ $s = t + 2$ —— (4)

(1) $\Rightarrow$ $3 + 2t = -2 + 3(t+2)$

$3 + 2t = -2 + 3t + 6$

$t = -1$

So, (4) $\Rightarrow$ $s = -1 + 2 = 1$.

Since we did not use (2):

L.H.S. $= 1 - (-1) = 2$

R.H.S. $= 3 - 1 = 2$

So, the lines intersect.

$x = 3 + (2)(-1) = 3 - 2 = 1$

$y = 1 - (-1) = 2$

$z = 4 + (-1) = 3$

Point of intersection $(1, 2, 3)$.

(36) Parametric Equations:

Line 1:	$x = 2t - 1$	$y = -t + 3$	$z = t + 2$
Line 2:	$x = 2s - 1$	$y = 3s - 1$	$z = -4s + 3$

$2t - 1 = 2s - 1$ —— (1)

$-t + 3 = 3s - 1$ —— (2)

$t + 2 = -4s + 3$ —— (3)

(1) $\Rightarrow 2t - \cancel{1} = 2s - \cancel{1} \Rightarrow t = s.$

(2) $\Rightarrow -t + 3 = 3t - 1 \Rightarrow 2 = 4t \Rightarrow t = \frac{1}{2} = s$

from (3) $\Rightarrow$ L.H.S. $\frac{1}{2} + 2 = \frac{5}{2}$

R.H.S.

$-4 \cdot \frac{1}{2} + 3 = -2 + 3 = 1$

So, L.H.S. $\neq$ R.H.S.

So, the two lines does not intersect.

(37) $\dfrac{x-3}{4} = \dfrac{y-1}{2} = \dfrac{z+1}{1}$.

$\underline{v} = 4\underline{i} + 2\underline{j} + 1\underline{k}$

$\|\underline{v}\| = \sqrt{4^2+2^2+1^2} = \sqrt{16+4+1} = \sqrt{21}$

Unit vectors $= \pm\dfrac{1}{\sqrt{21}}(4\underline{i} + 2\underline{i} + 1\underline{k})$

$\Rightarrow \dfrac{4}{\sqrt{21}}i + \dfrac{2}{\sqrt{21}}\underline{j} + \dfrac{1}{\sqrt{21}}k$

and $\dfrac{-4}{\sqrt{21}}\underline{i} + \dfrac{-2}{\sqrt{21}}\underline{j} + \dfrac{-1}{\sqrt{21}}\underline{k}$

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(38) $\dfrac{x-1}{2} = \dfrac{y+2}{4} = \dfrac{z+5}{1}$

$\underline{v} = 2\underline{i} + 4\underline{j} + 1\underline{k}$

$\|\underline{v}\| = \sqrt{2^2+4^2+1^2} = \sqrt{4+16+1} = \sqrt{21}$

Unit vectors:

$\dfrac{2}{\sqrt{21}}\underline{i} + \dfrac{4}{\sqrt{21}}\underline{j} + \dfrac{1}{\sqrt{21}}\underline{k}$

and $\dfrac{-2}{\sqrt{21}}\underline{i} + \dfrac{-4}{\sqrt{21}}\underline{j} + \dfrac{-1}{\sqrt{21}}\underline{k}$

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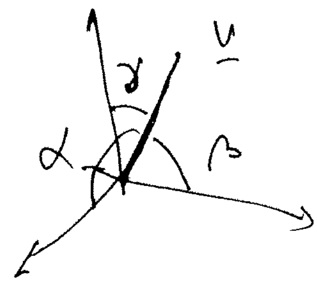
(31) $\underline{v} = 2\underline{i} - 3\underline{j} - 5\underline{k}$



$\|\underline{v}\| = \sqrt{2^2 + (-3)^2 + (-5)^2}$

$= \sqrt{4 + 9 + 25}$

$\|\underline{v}\| = \sqrt{38}$

$\cos\alpha = \dfrac{\underline{v} \cdot \underline{i}}{\|\underline{v}\|} = \dfrac{2}{\sqrt{38}} \Rightarrow \alpha \approx 71^\circ$

$\cos\beta = \dfrac{\underline{v} \cdot \underline{j}}{\|\underline{v}\|} = \dfrac{-3}{\sqrt{38}} \Rightarrow \beta \approx 119^\circ$

$\cos\gamma = \dfrac{\underline{v} \cdot \underline{k}}{\|\underline{v}\|} = \dfrac{-5}{\sqrt{38}} \Rightarrow \gamma \approx 144^\circ$

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(32) $\underline{v} = 3\underline{i} - 2\underline{k}$; $\|\underline{v}\| = \sqrt{3^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$

$\cos\alpha = \dfrac{\underline{v} \cdot \underline{i}}{\|\underline{v}\|} = \dfrac{3}{\sqrt{13}} \Rightarrow \alpha \approx 34^\circ$

$\cos\beta = \dfrac{\underline{v} \cdot \underline{j}}{\|\underline{v}\|} = 0 \Rightarrow \beta \approx 90^\circ$

$\cos\gamma = \dfrac{\underline{v} \cdot \underline{k}}{\|\underline{v}\|} = \dfrac{-2}{\sqrt{13}} \Rightarrow \gamma \approx 124^\circ$

(33) $\underline{v} = 5\underline{i} - 4\underline{j} + 3\underline{k}$

$\|\underline{v}\| = \sqrt{5^2 + (-4)^2 + 3^2} = \sqrt{25+16+9} = \sqrt{50}$

$\|\underline{v}\| = 5\sqrt{2}$

$\cos\alpha = \dfrac{\underline{v}\cdot\underline{i}}{\|\underline{v}\|} = \dfrac{5}{5\sqrt{2}} = \dfrac{1}{\sqrt{2}} \Rightarrow \alpha = 45^\circ$

~~Cos β~~ $\cos\beta = \dfrac{\underline{v}\cdot\underline{j}}{\|\underline{v}\|} = \dfrac{-4}{5\sqrt{2}} \Rightarrow \cancel{\alpha}\ \beta \approx 124^\circ$

$\cos\gamma = \dfrac{\underline{v}\cdot\underline{k}}{\|\underline{v}\|} = \dfrac{3}{5\sqrt{2}} \Rightarrow \gamma \approx 65^\circ$

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(34) $\underline{v} = \underline{j} - 5\underline{k} \Rightarrow \|\underline{v}\| = \sqrt{1 + (-5)^2} = \sqrt{26}$.

$\cos\alpha = \dfrac{\underline{v}\cdot\underline{i}}{\|\underline{v}\|} = 0 \Rightarrow \alpha = 90^\circ$

$\cos\beta = \dfrac{\underline{v}\cdot\underline{j}}{\|\underline{v}\|} = \dfrac{1}{\sqrt{26}} \Rightarrow \beta = 79^\circ$

$\cos\gamma = \dfrac{\underline{v}\cdot\underline{k}}{\|\underline{v}\|} = \dfrac{-5}{\sqrt{26}} \Rightarrow \gamma = 169^\circ$

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§ 9.6

(8) $P(-1, 3, 5)$; $\underline{N} = 2\underline{i} + 4\underline{j} - 3\underline{k}$.

plane: $2(x - (-1)) + 4(y - 3) - 3(z - 5) = 0$.

$\Rightarrow 2x + 4y - 3z + (2 - 12 + 15) = 0$

$\Rightarrow 2x + 4y - 3z + 5 = 0$

---

(9) $P(0, -7, 1)$; $\underline{N} = -\underline{i} + \underline{k}$

$-1(x - 0) + 0(y - (-7)) + 1(z - 1) = 0$

$-x + z - 1 = 0$

---

(14) $2x + 4y - 3z = 4$

$\underline{N} = 2\underline{i} + 4\underline{j} - 3\underline{k} \Rightarrow \|\underline{N}\| = \sqrt{2^2 + 4^2 + (-3)^2} = \sqrt{4 + 16 + 9}$

$\|\underline{N}\| = \sqrt{29}$

$$\pm \frac{\underline{N}}{\|\underline{N}\|} = \frac{\pm 2}{\sqrt{29}}\underline{i} + \frac{\pm 4}{\sqrt{29}}\underline{j} - \frac{\pm 3}{\sqrt{29}}$$

i.e. $$\frac{2}{\sqrt{29}}\underline{i} + \frac{4}{\sqrt{29}}\underline{j} - \frac{3}{\sqrt{29}}\underline{k}$$

and $$\frac{-2}{\sqrt{29}}\underline{i} - \frac{4}{\sqrt{29}}\underline{j} + \frac{3}{\sqrt{29}}\underline{k}$$

(15) $5x - 3y + 2z = 15$

$\underset{\sim}{N} = 5\underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k} \Rightarrow \|\underset{\sim}{N}\| = \sqrt{5^2 + (-3)^2 + 2^2} = \sqrt{25+9+4}$

$\|\underset{\sim}{N}\| = \sqrt{38}$

unit vectors:

$$\frac{5}{\sqrt{38}}\underset{\sim}{i} - \frac{3}{\sqrt{38}}\underset{\sim}{j} + \frac{2}{\sqrt{38}}\underset{\sim}{k}$$

and

$$\frac{-5}{\sqrt{38}}\underset{\sim}{i} + \frac{3}{\sqrt{38}}\underset{\sim}{j} - \frac{2}{\sqrt{38}}\underset{\sim}{k}$$

---

(18) $P(1,1,-1)$; $x - y + 2z = 4$

$\Rightarrow x - y + 2z - 4 = 0$.

$$\text{distance} = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2+B^2+C^2}} = \frac{|1 - 1 - 2 - 4|}{\sqrt{1^2 + (-1)^2 + 2^2}}$$

$$= \frac{6}{\sqrt{6}} = \sqrt{6}.$$

---

(19) $P(2,1,-2)$; $3x - 4y + z = -1 \Rightarrow 3x - 4y + z + 1 = 0$

$$\text{distance} = \frac{|(3)(2) - (4)(1) + (-2) + 1|}{\sqrt{3^2 + (-4)^2 + 1^2}} = \frac{|6 - 4 - 2 + 1|}{\sqrt{26}}$$

$$= \frac{1}{\sqrt{26}}$$

(20) $P(a,-a,2a)$; $2ax - y + az = 4a$; $a \neq 0$.

$$\Rightarrow 2ax - y + az - 4a = 0.$$

$$\text{distance} = \frac{|2a^2 + a + 2a^2 - 4a|}{\sqrt{4a^2+1+a^2}} = \frac{|4a^2 - 3a|}{\sqrt{5a^2+1}}$$

---

(21) $P(a,2a,3a)$; $3x - 2y + z = -\frac{1}{a}$ $a \neq 0$

$$3x - 2y + z + \frac{1}{a} = 0$$

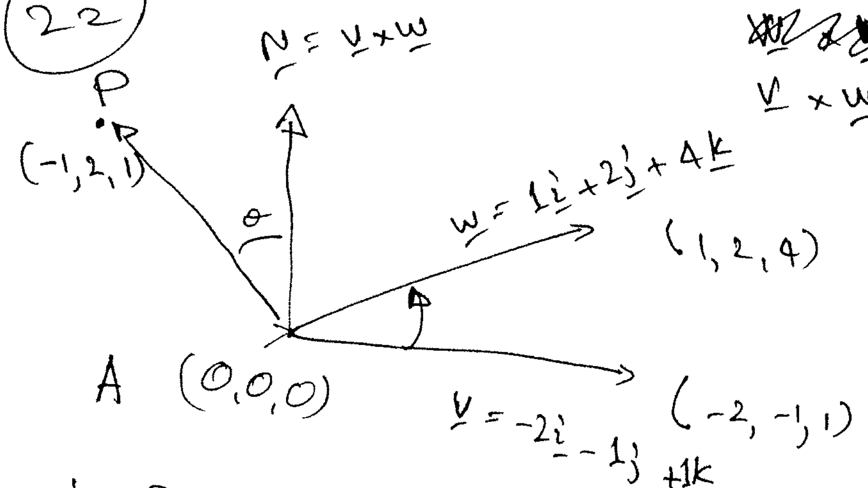
$$\text{distance} = \frac{|3a - 4a + 3a + \frac{1}{a}|}{\sqrt{9+4+1}} = \frac{|2a + \frac{1}{a}|}{\sqrt{14}} = \frac{|2a^2+1|}{|a|\sqrt{14}}$$

well, since $2a^2 + 1 > 0$,

$$\text{distance} = \frac{2a^2+1}{|a|\sqrt{14}}$$

---

(22)



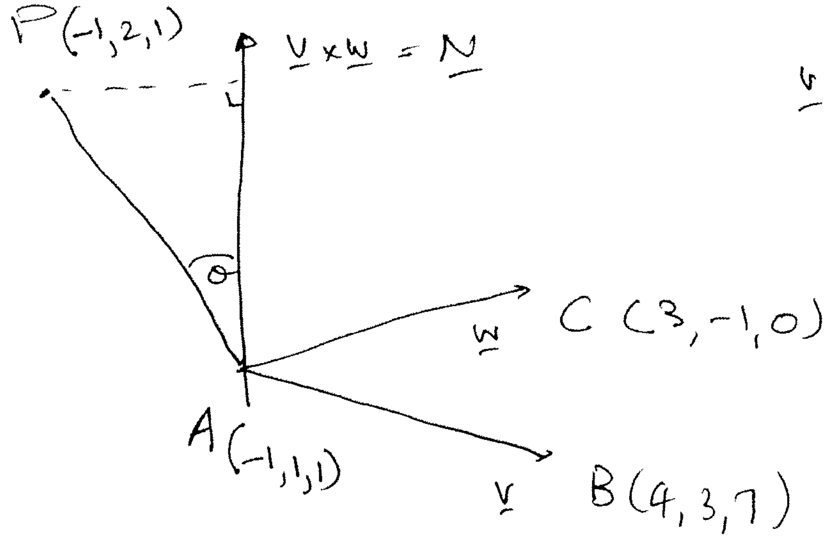
$$\underline{v} \times \underline{w} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -2 & -1 & 1 \\ 1 & 2 & 4 \end{vmatrix}$$

$$= (-6)\underline{i} + (7)\underline{j} + (-3)\underline{k}$$

$$\vec{AP} = -1\underline{i} + 2\underline{j} + 1\underline{k}$$

$$\frac{|\vec{AP} \cdot \underline{N}|}{\|N\|} = \frac{6 + 14 - 3}{\sqrt{36+49+9}} = \frac{17}{\sqrt{84}}$$

(23)



$\underline{v} = 5\underline{i} + 2\underline{j} + 6\underline{k}$

$\underline{w} = 4\underline{i} - 2\underline{j} - 1\underline{k}$

$\overrightarrow{AP} = 0\underline{i} + 1\underline{j} + 0\underline{k}$

$$\underline{N} = \underline{v} \times \underline{w} = \begin{vmatrix} i & j & k \\ 5 & 2 & 6 \\ 4 & -2 & -1 \end{vmatrix} = \cancel{40i\ 2j}$$

$$= 10\underline{i} + 29\underline{j} - 18\underline{k}$$

$$\|\underline{N}\| = \|\underline{v} \times \underline{w}\| = \sqrt{10^2 + 29^2 + (18)^2}$$

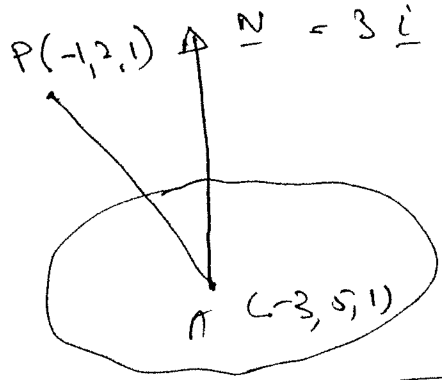
so, distance $= \dfrac{|N \cdot \overrightarrow{AP}|}{\|\underline{N}\|} = \dfrac{29}{\sqrt{1265}}$

$29^2 = (30-1)^2$
$= 900 - 60 + 1$
$18^2 = (20-2)^2$
$= 400 - 80 + 4$
$10^2 = 100$
$1400 - 140 + 5$
$= 1265$

College Algebra in action

(24)

$\underline{N} = 3\underline{i} + \underline{j} + 5\underline{k}$.



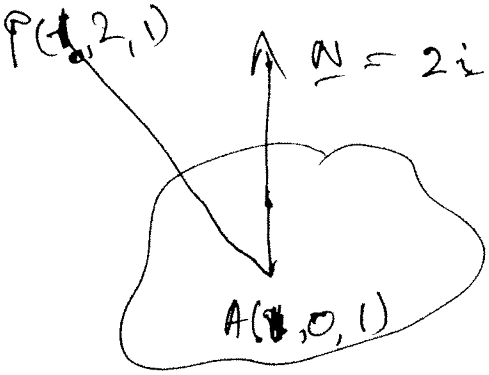
$\overrightarrow{AP} = 2\underline{i} - 3\underline{j} + 0\underline{k}$

distance $= \dfrac{|\overrightarrow{AP} \cdot N|}{\|\underline{N}\|} = \dfrac{3}{\sqrt{35}}$

$\|\underline{N}\| = \sqrt{3^2 + 1^2 + 5^2} = \sqrt{9 + 1 + 25}$
$= \sqrt{35}$

$\overrightarrow{AP} \cdot \underline{N} = 6 - 3 = 3$

25



$\underline{N} = 2\underline{i} - \underline{j} + 2\underline{k}$

$\|\underline{N}\| = \sqrt{2^2 + (-1)^2 + 2^2}$

$= \sqrt{4 + 1 + 4} = \sqrt{9}$

$= 3$

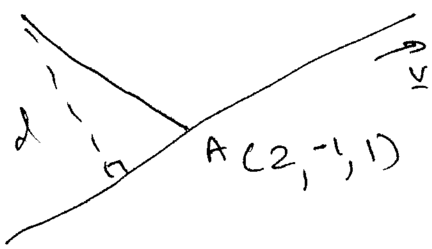
$\vec{AP} = -2\underline{i} + 2\underline{j} + 0\underline{k}$

$\vec{AP} \cdot \vec{N} = \underline{N} \cdot \vec{AP} = -4 - 2 = -6.$

distance $= \dfrac{|\vec{AP} \cdot N|}{\|\underline{N}\|} = \dfrac{|-6|}{\sqrt{9}} = \dfrac{6}{3} = 2.$

---

29 P (1, 0, -1)



$\vec{v} = 3\underline{i} + 1\underline{j} + 2\underline{k}$

$\vec{AP} = -1\underline{i} + 1\underline{j} - 2\underline{k}$

$\vec{AP} \times \underline{u} = \begin{vmatrix} i & j & k \\ -1 & 1 & -2 \\ 3 & 1 & 2 \end{vmatrix}$

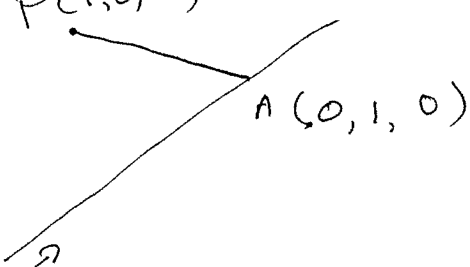
$= 4\underline{i} - 4\underline{j} - 4\underline{k}$

$\|\underline{v}\| = \sqrt{3^2 + 1^2 + 2^2} = \sqrt{14}.$

distance, $d = \dfrac{\|\vec{AP} \times \underline{v}\|}{\|\underline{v}\|} = \dfrac{4\sqrt{3}}{\sqrt{14}}$

---

30 P(1,0,-1)



$\vec{\underline{v}} = 3\underline{i} + 2\underline{j} + 1\underline{k}$

$\vec{AP} = 1\underline{i} - 1\underline{j} - 1\underline{k}$

$\vec{AP} \times \underline{v} = \begin{vmatrix} i & j & k \\ 1 & -1 & -1 \\ 3 & 2 & 1 \end{vmatrix}$

$= 1\underline{i} - 4\underline{j} + 5\underline{k}$

$\|\vec{AP} \times \underline{v}\| = \sqrt{1^2 + (4)^2 + 5^2} = \sqrt{1 + 16 + 25} = \sqrt{42}$

$\|\underline{v}\| = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$

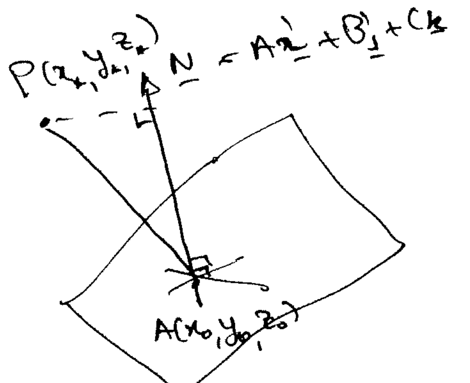
$d = \dfrac{\|\vec{AP} \times \underline{v}\|}{\|\underline{v}\|} = \dfrac{\sqrt{42}}{\sqrt{14}} = \sqrt{3}$ $\longrightarrow$ $42 = 14 \times 3$

26, 27, 28 were not assigned but I will provide the solutions:

Note: Lines in $\mathbb{R}^2$ are similar to planes in $\mathbb{R}^3$ in many ways. In particular in distance calculations.

So, the distance from a point to a line in $\mathbb{R}^2$ can be found in a way similar to the distance calculation from a point to a plane in $\mathbb{R}^3$. Namely:

In $\mathbb{R}^3$. Point & Plane

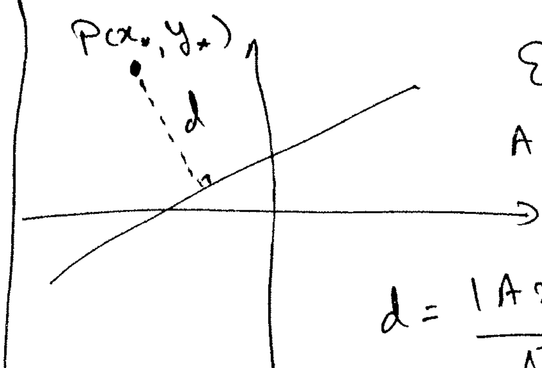


or Eq. of plane
$$Ax + By + Cz + D = 0.$$

So,
$$d = \frac{|Ax_* + By_* + Cz_* + D|}{\sqrt{A^2 + B^2 + C^2}}$$

or $$d = \frac{|\overrightarrow{AP} \cdot \underline{N}|}{\|\underline{N}\|}$$

In $\mathbb{R}^2$ Point & Line



Eq. of line
$$Ax + By + C = 0$$

$$d = \frac{|Ax_* + By_* + C|}{\sqrt{A^2 + B^2}}.$$

(This is probably the easiest here!).

We can write the vector form still it will be

$$\frac{|\overrightarrow{AP} \cdot \underline{N}|}{|\underline{N}|}$$ if normal is given

$\boxed{\text{or}}$ $$\frac{\overrightarrow{AP} \times \underline{V}}{\|\underline{V}\|}$$ if $\underline{V}$ is given

26

~~distance~~ line : $3x - 4y + 8 = 0$

Point : $(4, 5)$

distance : $\dfrac{|\overset{12}{(3)(4)} - \overset{20}{(4)(5)} + 8|}{\sqrt{4^2 + 3^2}} = \dfrac{0}{\sqrt{25}} = 0$

i.e. the point $(4,5)$ is on the line.

27

Line: $3x - 4y + 8$, Point $(9, -3)$

distance: $\dfrac{|\overset{27}{(3)(9)} - \overset{12}{(4)(-3)} + 8|}{\sqrt{4^2 + 3^2}} = \dfrac{47}{\sqrt{25}}$

distance $= \dfrac{47}{5}$

28

Point $(4, -3)$, line: $12x + 5y - 2 = 0$.

distance $= \dfrac{|(12)(4) + (5)(-3) - 2|}{\sqrt{12^2 + 5^2}}$

$= \dfrac{|48 - 15 - 2|}{\sqrt{144 + 25}} = \dfrac{|31|}{\sqrt{169}}$

$= \dfrac{31}{13}$

(3/4) Whenever you are asked to find the distance between 2 lines, you should see if the are parallel or skew!

---

(32) $\dfrac{x+1}{3} = \dfrac{y-2}{-2} = \dfrac{z-1}{1}$ & $\dfrac{x-2}{5} = \dfrac{y+1}{1} = \dfrac{z}{3}$

$\underline{v}_1 = 3\underline{i} - 2\underline{j} + 1\underline{k}$ $\quad\quad$ $\underline{v}_2 = 5\underline{i} + 1\underline{j} + 3\underline{k}$

clearly, $\underline{v}_1$ and $\underline{v}_2$ are not parallel.

So, find, ~~A~~ we can ~~ya~~ use the distance formula I gave:

$A_1 = -1\underline{i} + 2\underline{j} + 1\underline{k}$ $\quad\quad$ $A_2 = 2\underline{i} - 1\underline{j} + 0\underline{k}$

So, $\overrightarrow{A_1A_2} = 3\underline{i} - 3\underline{j} - 1\underline{k}$

~~An~~ $\underline{v}_1 \times \underline{v}_2 = \begin{vmatrix} i & j & k \\ 3 & -2 & 1 \\ 5 & 1 & 3 \end{vmatrix} = -7\underline{i} - 4\underline{j} + 13\underline{k}$

$\overrightarrow{A_1A_2} \cdot (\underline{v}_1 \times \underline{v}_2) = -21 + 12 - 13 = -26.$

$\|\underline{v}_1 \times \underline{v}_2\| = \sqrt{(-7)^2 + (-4)^2 + 13^2} = \sqrt{49 + 16 + 169} = \sqrt{234}.$

$\therefore$ distance $= \dfrac{|\overrightarrow{A_1A_2} \cdot (\underline{v}_1 \times \underline{v}_2)|}{\|\underline{v}_1 \times \underline{v}_2\|} = \dfrac{|-26|}{\sqrt{234}} = \dfrac{26}{\sqrt{234}}$

(33) $x = 2 - t,\ y = 5 + 2t;\ z = 3t$ } line 1

$\Rightarrow A_1 = (2, 5, 0);\quad \underline{v}_1 = -1\underline{i} + 2\underline{j} + 3\underline{k}$

$x = 2t;\ y = -1 - t;\ z = 1 + 2t$ } line 2

$\Rightarrow A_2 = (0, -1, 1);\quad \underline{v}_2 = 2\underline{i} - 1\underline{j} + 2\underline{k}$

Clearly they are not parallel:

Easy way to check if the vectors $\underline{v}_1$ & $\underline{v}_2$ are scaled versions: check the ratios of the corresponding terms:

If $\underline{v}_1 = a_1\underline{i} + b_1\underline{j} + c_1\underline{k}$ &

$\underline{v}_2 = a_2\underline{i} + b_2\underline{j} + c_2\underline{k}$

then, they are <u>parallel if</u>

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Clearly in this problem, $\frac{-1}{2} \neq \frac{2}{-1} \neq \frac{3}{2}$

$$\underline{v}_1 \times \underline{v}_2 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -1 & 2 & 3 \\ 2 & -1 & 2 \end{vmatrix} = 7\underline{i} + 8\underline{j} - 3\underline{k}$$

$\overrightarrow{A_1A_2} = -2\underline{i} - 6\underline{j} + 1\underline{k}$; $\overrightarrow{A_1A_2} \cdot (\underline{v}_1 \times \underline{v}_2) = -14 - 48 - 3$

$\|\underline{v}_1 \times \underline{v}_2\| = \sqrt{49 + 64 + 9} = \sqrt{122}$ $\Rightarrow$ distance $= \frac{65}{\sqrt{122}}$

(34) $\dfrac{x+1}{1} = \dfrac{y-3}{2} = \dfrac{z+2}{3}$

$\Rightarrow A_1 = -1\underline{i} + 3\underline{j} - 2\underline{k}$

$\underline{v}_1 = 1\underline{i} + 2\underline{j} + 3\underline{k}$

line 1.

Line 2: Passes through $(1,3,-2)$ & $(0,1,-1)$.

so, take $A_2 = 1\underline{i} + 3\underline{j} - 2\underline{k}$

$\underline{v}_2 = (0-1)\underline{i} + (1-3)\underline{j} + (-1-(-2))\underline{k}$

$= -1\underline{i} - 2\underline{j} + 1\underline{k}$

so, $\underline{v}_1 \times \underline{v}_2 = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ -1 & -2 & 1 \end{vmatrix} = 8\underline{i} - 4\underline{j} + 0\underline{k}$

$\overrightarrow{A_1A_2} = -2\underline{i}$

so, $\dfrac{|\overrightarrow{A_1A_2} \cdot (\underline{v}_1 \times \underline{v}_2)|}{\|\underline{v}_1 \times \underline{v}_2\|} = \dfrac{|-16|}{\sqrt{64+16}} = \dfrac{16}{\sqrt{80}}$

$80 = 16 \times 5$

so distance $= \dfrac{\sqrt{16}}{\sqrt{5}}$

(37) $\frac{x-1}{3} = \frac{y}{-2} = \frac{z+1}{1} \Rightarrow$ $A_1 = 1\underline{i} + 0\underline{j} - 1\underline{k}$

$\underline{v} = 3\underline{i} - 2\underline{j} + 1\underline{k}$

$x + 2y + z - 1 = 0$

$x + 2y + z = 1 \Rightarrow \underline{N} = 1\underline{i} + 2\underline{j} + 1\underline{k}$

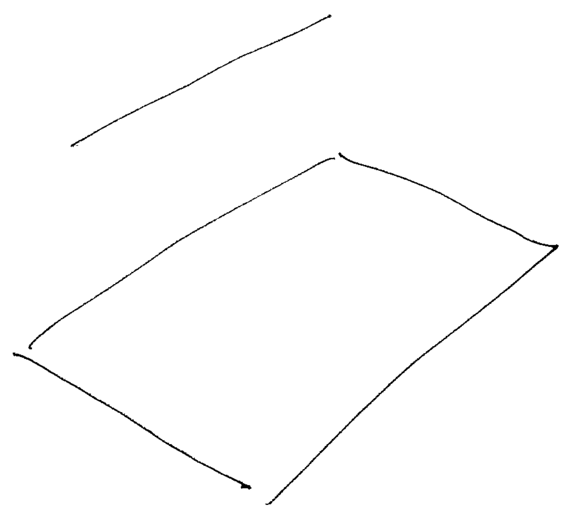
(a)

$\underline{v} \cdot \underline{N} = 3 - 4 + 1 = 0$

So, $\underline{v}$ is perpendicular to $\underline{N}$.

$\therefore$ The given line & plane are parallel.

(b)



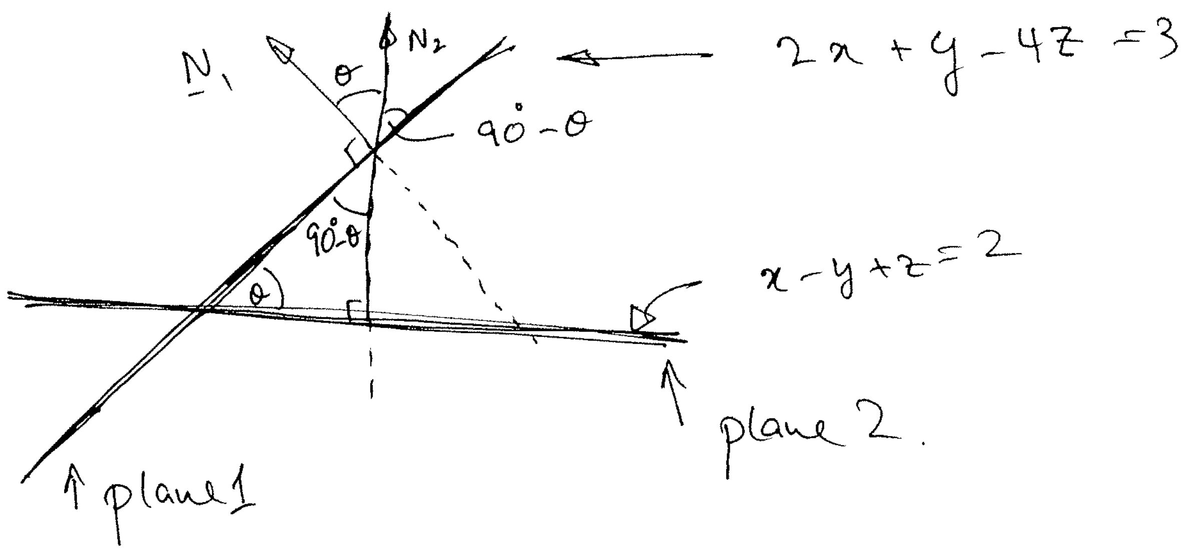
distance:

(since the line is parallel to the plane, ~~you~~ ~~can~~ the distance from any point on the line to the plane will be the same.

so, pick $A_1 (1, 0, -1)$.

distance: $\frac{|(1) + (2)(0) + (-1) - 1|}{\sqrt{1^2 + 2^2 + 1^2}} = \frac{1}{\sqrt{6}}$

(54) The angle between two planes is defined to be the acute angle between their normal vectors:



$\underline{N}_1 = 2\underline{i} + 1\underline{j} - 4\underline{k}$; $\underline{N}_2 = 1\underline{i} - 1\underline{j} + 1\underline{k}$

$\underline{N}_1 \cdot \underline{N}_2 = 2 - 1 - 4 = -3. = \|N_1\| \, \|N_2\| \cos\theta.$

$\|N_1\| = \sqrt{2^2 + 1^2 + (-4)^2} = \sqrt{4 + 1 + 16} = \sqrt{21} = \sqrt{(7)(3)}$

$\|\underline{N}_2\| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}.$

so, $\|N_1\| \, \|N_2\| = \sqrt{21} \cdot \sqrt{3} = 3\sqrt{7}$

$\cos\theta = \dfrac{\underline{N}_1 \cdot \underline{N}_2}{\|\underline{N}_1\| \|\underline{N}_2\|} = \dfrac{-3}{3\sqrt{7}} = \dfrac{-1}{\sqrt{7}} \Rightarrow \theta \approx 112^\circ$

$\therefore$ Acute angle $= 180^\circ - 112^\circ \approx 68^\circ$